Answers to Relative Rates of Growth Homework

For each pair of functions given, determine which one (if either) grows faster as \( x \to \infty \). You must give detailed arguments using limits for \#2c,d,e and \#3a,b,c,e,g. For each of the others you may use limits or just explain your reasoning with a sentence. Try to notice patterns while you do this!

1.a) \( x^2, \ 2 - \sqrt{x} + 4x^2 \) same growth rate

b) \( x^2, \ \sin(x^3) + x^2 \) same growth rate

c) \( 100x^2, \ 2 - \sqrt{x} + 4x^3 \) \( 2 - \sqrt{x} + 4x^3 \) grows faster

d) \( 100x^2, \ 2x + x^2 \) \( 2x + x^2 \) grows faster

2.a) \( x^{100}, \ e^x \) \( e^x \) grows faster

b) \( 2^x, \ e^x \) \( e^x \) grows faster

c) \( 2^x, \ e^{-x} \) \( 2^x \) grows faster

d) \( 2^{2x}, \ e^x \) \( 2^{2x} \) grows faster

e) \( x2^x, \ e^x \) \( e^x \) grows faster

f) \( x, \ e^{\cos x} \) \( x \) grows faster

3.a) \( \ln x, \ \log_2 x \) same growth rate

b) \( \ln x, \ x \) \( x \) grows faster

c) \( \ln x, \ \sqrt[10]{x} \) \( \sqrt[10]{x} \) grows faster

d) \( \ln x, \ \cos \ln x \) \( \ln x \) grows faster

e) \( \ln x, \ \ln x^2 \) same growth rate

f) \( \ln x, \ (\ln x)^2 \) \( (\ln x)^2 \) grows faster

g) \( \ln x, \ \ln(\ln x) \) \( \ln x \) grows faster