Math 527 - Metric and Topological Spaces


Syllabus


2. Open and closed sets: interior, closure, boundary and their properties. Relatively open sets in a subspace.

3. Continuous maps: the $\varepsilon$-$\delta$ definition, relation to open sets. Convergence of sequences in a metric space: continuity and sequential continuity. Homeomorphisms.

4. Compactness and sequential compactness. Compact sets are closed and bounded. A closed subset of a compact space is compact. The Heine-Borel theorem. The continuous image of a compact space is compact. The Lebesgue covering theorem.

5. Connectedness and path connectedness. Equivalence for locally path connected spaces, e.g. open subsets of $\mathbb{R}^n$. Preservation of connectedness under continuous maps. Connected subsets of $\mathbb{R}$ are intervals. The intermediate value theorem.

6. Cauchy sequences, completeness, precompactness, uniform continuity. Completeness of $\mathbb{R}^n$, completeness of $C(X)$ where $X$ is compact.

7. The Banach contraction mapping principle. The Baire category theorem. Simple applications (e.g. the real numbers are uncountable, there exists a continuous function which is nowhere differentiable).


10. The Hausdorff property. Compactness, connectedness (recapitulation). Compact subspaces are closed in a Hausdorff space. A continuous bijection from a compact space to a Hausdorff space is a homeomorphism.

11. First and second countability. Examples distinguishing “sequential” from general concepts (e.g. sequential compactness vs. compactness, sequential continuity vs. continuity)

13. Quotient spaces, quotient topologies. The universal property of the quotient topology. Simple sufficient conditions for the quotient topology to be Hausdorff. Examples, including “attaching” or “gluing” spaces, “collapsing” a subspace to a point, cones, suspensions, projective spaces.


15. The fundamental group: definition, functorial properties. The homotopy lifting property for the covering map $\mathbb{R} \mapsto S^1$ Computation of $\pi_1(S^1)$ and simple applications, such as the Brouwer fixed point theorem in 2 dimensions.

References


