Workshop “Frontiers of Interdisciplinary Mathematics”

Penn State University, 114 McAllister Building, May 9 – 11, 2017.

Sponsored by the *Center for Interdisciplinary Mathematics* and by the *Center for Mathematics of Living and Mimetic Matter*, Penn State University.

Program

Tuesday, May 9.

9:00 - 9:10am *Opening remarks*

9:10 - 10:10am **Suncica Canic** (University of Houston). Fluid-composite structure interaction and blood flow.

10:10 - 10:30am *Coffee break*

10:30 - 11:30am **Rinaldo Colombo** (University of Brescia) Conservation laws in vehicular traffic and crowd dynamics.

11:30 - 12:30pm *CONTRIBUTED TALKS*

11:30 - 12:00 – **Michele Palladino** (Penn State) Growth models for tree stems and vines.

12:00 - 12:30 - **Matt Mizuhara** (Penn State) Analysis and simulation of minimal models of crawling cell motility.

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12:30 - 2:30pm lunch break

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2:30-3:30pm **Erich Walter Farkas** (University of Zürich and ETH Zürich and Swiss Finance Institute). The mathematics of financial risk management: approach, challenges and recent developments.

3:30 - 4:00pm *CONTRIBUTED TALK*

3:30-4:00 - **Khai Nguyen** (North Carolina State University, Raleigh)

A stochastic model of optimal debt management and bankruptcy.

4:00 - 4:20pm *Coffee break*

4:20 - 6:20pm *CONTRIBUTED TALKS*

4:20-4:50 - **Yilun Jiang** (Penn State) Optimal strategies for a debt management problem.

4:50-5:20 - **Hongxu Wei** (Penn State) A dynamic model of the limit order book.

5:20-5:50 - **Luca DiPersio** (U. of Verona, Italy) Stochastic models for wind energy markets.

5:50-6:20 - **Luca Prezioso** (U. of Verona, Italy) Optimal stochastic control for an asset-liability model with takeover opportunities.

6:30 - 8:00pm *Reception/dinner with poster presentations*

POSTER PRESENTATIONS:

- **Hai Chi** (Penn State) Poiseuille like flow in trapezoidal nozzle, computation and analysis.

- **Priyanka Patki** (Penn State) Poro-elastic properties via homogenization.

- **Diego Ricciotti** (U. of Pittsburgh) Plates with incompatible prestrain of higher order.

- **Elena Rossi** (U. of Brescia, Italy) Hyperbolic predators vs. parabolic prey: analysis, numerics and open problems.

- **Siddarth Sharma** (U. of Maryland, College Park)

Wednesday, May 10.

9:00 - 10:00am **Igor Aronson** (Penn State) Dynamic colloidal materials: ferromagnetic swimmers, spinners and rollers.

10:00 - 11:00am **Marta Lewicka** (U. of Pittsburgh) A model of controlled growth.

11:00 - 11:20am *Coffee break*

11:20 - 12:50 *CONTRIBUTED TALKS*

11:20-11:50 - **Mykhailo Potomkin** (Penn State) Kinetic PDE approach to model active ink.

11:50-12:20 - **Yuliya Gorb** (U. of Houston) Efficient approximations for high contrast composite materials.

12:20–12:50 - **Xiaojie Wu** (Penn State) Multiscale simulations of crystalline solids using atomistic-based boundary element method.

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12:50 - 2:30pm lunch break

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2:30 - 3:30pm **Doron Levy** (University of Maryland, College Park) The role of the autologous immune response in chronic myelogenous leukemia.

3:30 - 4:30pm *CONTRIBUTED TALKS*

3:30 - 4:00 - **Nastassia Pouradier Duteil** (Rutgers Camden) Developmental PDEs.

4:00 - 4:30 - **Ryan Murray** (Penn State) Optimal transportation, economics and gerrymandering.

4:30 - 4:50pm *Coffee break*

4:50 - 6:20pm *CONTRIBUTED TALKS*

4:50 - 5:20 - **Giulia Cavagnari** (Rutgers Camden) Control problems in the space of measures.

5:20 - 5:50 - **Alex Misiats** Convex duality in nonconvex variational problems.

5:50 - 6:20 - **Weiqi Chu** (Penn State) A fluctuating energy transport model for heat conduction problems.

Thursday, May 11.

9:00 - 10:00am **Ken Kamrin** (Massachusetts Institute of Technology) Advancing mathematical models for deforming materials.

10:00 - 10:30am *CONTRIBUTED TALK*

10:00 - **Misha Genkin** (Penn State) Interaction of topological defects and normal inclusions in a model of living nematic.

10:30 - 10:50am *Coffee break*

10:50 - 12:20 *CONTRIBUTED TALKS*

10:50-11:20 - **Francesco De Anna** (Penn State) Global well-posedness for some nematic liquid crystals models.

11:20-11:50 - **Robby Creese** (Penn State) A new approach to computing correlations in many particle systems.

11:50-12:20 - **Yifan Wang** (U. of Houston) Discontinuous Galerkin arbitrary Lagrangian Eulerian partitioned approach to solving fluid-structure interaction problems with incompressible, viscous fluids and elastic structures.

12:20pm *Closing remarks.*

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Abstracts

Igor Aronson - *Dynamic colloidal materials: ferromagnetic swimmers, spinners and rollers.*

Assemblages of microscopic colloidal particles exhibit fascinating collective motion when energized by electric of magnetic fields. The dynamic states range from coherent vortical motion to phase separation and dynamic self-assembly. While colloidal systems are relatively simple, their understanding, especially in out of equilibrium conditions, remains elusive. Here our experimental and computational studies of large-scale collective behavior emerging in a system of ferromagnetic colloidal particles energized by a uniaxial alternating magnetic field will be surveyed. Observed dynamic states include self-assembled microswimmers, spinners, and wires. We also report on the emergence of flocking behavior and global rotation and synchronization in the system of rolling ferromagnetic microparticles. To capture the complexity of observed phenomena, a hierarchy of mathematical models is developed, from coupled nonlinear ODEs for the positions and orientations of individual colloidal particles to hybrid ODE-PDE systems coupling particle motion to self-induced hydrodynamic flows. Analytical solutions and large-scale computational studies of mathematical models provide insights into emerging collective behavior and often lead to testable predictions. Our findings shed light on the onset of spatial coherence and synchronization in a large class of active systems, both synthetic (colloids, swarms of robots, biopolymers) and living (suspensions of bacteria, cell colonies, bird flocks).

Suncica Canic - *Fluid-composite structure interaction and blood flow.*

Composite structures are structures that are made of one or more constituent materials which, when combined, provide a structure with superior qualities with respect to either of the constituent components. Examples include laminated, multi-layered structures, and mesh-supported structures. They are ubiquitous in nature, biology, engineering, and technology. Human arterial walls, e.g., are composite structures made of several layers, each with different mechanical characteristics and thickness. They interact with blood flow by stretching and contracting, thereby acting as a secondary pump to the heart muscle. Understanding the interaction between such composite structures and flows of incompressible, viscous fluids is important for virtually all areas of science and engineering No mathematical results exist so far that analyze existence of solutions to nonlinear, fluid-structure interaction problems in which the structure is composed of several layers. In this talk we will summarize the main difficulties in studying this class of problems, and present a computational scheme based on which a proof of the existence of a weak solution was obtained. Our results reveal a new physical regularizing mechanism in FSI problems: inertia of the thin fluidstructure interface with mass regularizes evolution of FSI solutions. Implications of our theoretical results on modeling the human cardiovascular system will be discussed. This is a joint work with Boris Muha (University of Zagreb, Croatia), Martina Bukac (U of Notre Dame, US) and Roland Glowinski (UH). Numerical results with vascular stents were obtained with S. Deparis and D. Forti (EPFL, Switzerland). Collaboration with medical doctors Dr. S. Little (Methodist Hospital Houston) and Dr. Z. Krajcer (Texas Heart Institute) is also acknowledged.

Rinaldo M. Colombo - *Conservation laws in vehicular traffic and crowd dynamics.*

Conservation Laws are a class of partial differential equations whose general analytic theory is at present still far from being completely well established. Nevertheless, they allow the description of several phenomena, providing useful tools in coping with problems like traffic forecasts or the minimization of the evacuation time from closed environments. The present talk overviews recent results in this area, trying to underline the interplay between theoretical results and the use of computers.

Robby Creese - *A new approach to computing correlations in many particle systems.*

Abstract: We consider a system with a large number of interacting particles and random initial conditions. Direct simulations for such systems are computationally expensive due to the large number of particles and many realizations of random initial conditions. In this talk, a truncation of the BBGKY hierarchy recently introduced in [Berlyand et. al., SIAM JUQ 2016] will be described. Results of numerical simulations for various interacting potentials and comparison with the classical Kirkwood Superposition Approximation and the Mean Field Approximation will be presented.

Luca Di Persio - *Stochastic models for wind energy markets.*

Recent years have seen an impressive growth of the interest in production of wind energy as one of the most efficient renewable energy source. Unfortunately, such type of energy can be generated only in presence of a sufficient amount of wind, with suitable characteristics. Indeed, to optimize production, one has to avoid possible damages caused, for example, by wind storms, and take into account the direction and the constancy of the wind. In this talk we analyze some stochastic models of a wind power plant. We provide an analytical formula for the income generated by a wind mill, as a product of the electricity spot price times the amount of energy produced. According to concrete situations, these will be considered as two negatively correlated processes. Moreover, by means of some data sets, we also discuss possible approaches for the calibration of these models. Furthermore, we provide a basis for a rigorous study of quanto options in wind energy markets, by constructing a simple European-call type quanto option based on two indices, namely the spot price and an indicator of the wear that a power plant suffers during the years as a consequence of extreme weather events.

Walter Farkas - *The mathematics of financial risk management: approach, challenges and recent developments.*

Risks and their quantification are essential for the functioning of financial institutions: liability holders and regulators (on their behalf) are concerned how much risk capital an institution should hold such that unexpected losses can be absorbed. In this talk we will present the mathematical framework around this question, we will discuss the so-called risk measures and will address challenges of these concepts from both an economic and mathematical perspective.

Yilun Jiang - *Optimal strategies for a debt management problem.*

We study optimal strategies for a borrower who needs to repay his debt, in an infinite time horizon. An instantaneous bankruptcy risk is present, which increases with the size of the debt. This induces a pool of risk-neutral investors to offer discounted price for the bond, to compensate for the possible loss of their investments. Open-loop strategies are interpreted as Stackelberg equilibria, where the borrower announces his repayment strategy at all future times, and investors adjust the bond price accordingly. This yields a highly nonstandard optimal control problem, where the instantaneous dynamics depends on the entire future evolution of the system. We show the existence of optimal open-loop controls and derive necessary conditions for optimality. Feedback strategies are obtained via vanishing viscosity limit. For the stochastic model with positive diffusion, the existence of an equilibrium solution is obtained by a topological argument. Our analysis shows that under suitable assumptions, as the diffusion parameter approaches zero, this (possibly discontinuous) limit can be interpreted as an equilibrium solution to a non-cooperative differential game with deterministic dynamics.

Ken Kamrin - *Advancing mathematical models for deforming materials.*

The study of deforming solids, fluids, and other phases of matter has had a historical home within applied mathematics, which has over time spread into a number of engineering and physics disciplines. There remain many fundamental applied mathematics challenges within this broad topic. In fact, important open issues touch upon the quintessential tools of the applied mathematician: model extraction, analytical methods, and numerical methods. This talk will survey various open areas within continuum modeling of materials, highlighting these three important applied mathematics aspects. Emphasis will be placed on where the current state of the art is and areas for future improvements.

Marta Lewicka - *A model of controlled growth.*

In this talk, we will discuss a free boundary problem modeling the growth of a biological tissue. Our system consists of several PDEs for the evolution of the morphogen (the chemical controlling volume growth), coupled with the variational problem where the geometric shape of the growing tissue is determined by the instantaneous minimization of an elastic deformation energy, subject to a volumetric growth constraint. We show the local existence and uniqueness of a classical solution to this coupled PDE-variational model, up to a rigid motion. We further discuss the possible generalizations and modeling questions regarding growth of structures of positive codimension and give an overview of the related problems in the description of nonlinear elastic prestrained materials. This is a joint work with Alberto Bressan.

Doron Levy - *The role of the autologous immune response in chronic myelogenous leukemia.* Tyrosine kinase inhibitors such as imatinib (IM), have significantly improved treatment of chronic myelogenous leukemia (CML). Yet, most patients are not cured for undetermined reasons. In this talk we will describe our recent work on modeling the autologous immune response to CML. We will also discuss our previous results on cancer vaccines, drug resistance, and the dynamics of hematopoietic stem cells.

Alex Misiats - *Convex Duality in Nonconvex Variational Problems.*

We consider the minimization problem which models martensitic (diffusionless) phase transitions in shape memory alloys. Physical experiments suggest that if opposite phases are present at opposite sides of a rectangular sample, the transition has a form of a zig-zag wall. The mathematical model of this phenomenon involves the minimization of singularly perturbed elastic energy with phase constraints. By means of sharp upper and lower bounds, we show that the experimentally observed zig-zag structure provides optimal energy scaling law. Despite the fact that the problem is highly nonconvex due to the presence of phase constraints and the singular perturbation, in my talk, I will describe a relaxation method which allows to use the convex duality technique for the purpose of obtaining a sharp lower bound.

Ryan Murray - *Optimal transportation, economics and gerrymandering.*

In the United States political boundaries (e.g. congressional maps) are commonly drawn by elected officials. Political mischief often stems from this practice, which is known as gerrymandering. This talk will discuss how one can use optimal transportation to draw political boundaries using a transparent, algorithmic method. An introduction to optimal transportation, and computation of the same, will also be discussed.

Khai Nguyen - *A stochastic model of optimal debt management and bankruptcy.*

Consider a problem of optimal debt management which is modeled as a non-cooperative game between a borrower and a pool of risk neutral lenders. Since the debtor may go bankrupt, lenders charge a higher interest rate to offset the possible loss of part of their investment. In this talk, I will present results on existence and properties of optimal strategies, both in a deterministic and in a stochastic framework

Michele Palladino - *Growth models for tree stems and vines.*

In this talk, we propose a model describing the growth of tree stems and vine, taking into account also the presence of external obstacles. The system evolution is described by an integro-differential equation which becomes discontinuous when the stem hits the obstacle. The stem feels the obstacle reaction not just at the tip, but along the whole stem. This fact represents one of the main challenges to overcome, since it produces a cone of possible reactions which is not normal with respect to the obstacle. However, using the geometric structure of the problem and nonlinear analysis tools, we are able to prove existence and uniqueness of the solution under natural assumptions on the initial data.

Luca Prezioso - *Optimal stochastic control for an asset-liability model with takeover opportunities.*

In this talk we show how to derive the optimal control on the dividend and takeover policy of a firm. We consider a dynamic model for the asset and the liabilities of a Company, allowing the manager to accept external investment/divestment proposal, whose arrival times follow a Cox process. The manager decides if the firm should search for takeover/handover opportunities or not, and the acceptance of proposals, as well as the switch from a search regime to another, determining how his policy affects the Company's liabilities. The optimization problem consists in maximizing the discounted value of the dividends paid between the actual time and bankruptcy defined as the first time when the asset value goes under the firm liability value. The problem is formulated as a bi-dimensional singular switching control problem. Exploiting the dynamic programming principle, we derive the HJB equation associated to the problem and prove regularity properties for the solution, such as continuity and linear growth. A characterization of the solution to the optimal control problem, by mean of the viscosity approach, is also provided.

Honxu Wei - *A Dynamic Model of the Limit Order Book.*

A limit order book in a stock market is modeled as a noncooperative game for several players. An external buyer asks for a random amount $X > 0$ of a given asset. This amount will be bought at the lowest available price, as long as the price does not exceed a upper bound $\overline{P}$. One or more sellers offer various quantities of the asset at different prices, competing to fulfill the incoming order. In this talk, we suppose the size X of the order is not known a priori, and thus regarded as random variables. As for the maximum acceptable price P, we consider two cases, deterministic or random. Similarly, an external buy order can be considered. In this setting, we find conditions for the existence of a unique Nash equilibrium, where each player optimally prices his assets in order to maximize his own expected profit. We also consider an evolutionary model, involving a sequence of N incoming orders. Each can be either a buy order or a sell order. To simplify the analysis, we consider the limiting case with infinitely many players, each player holding a small amount of asset. We discuss how the shape of the limit order book varies in time, and the expected payoff of the various agents.