MATH312, REVIEW

1. Real numbers and monotone sequences.
   a) Completeness property for monotone bounded sequences.
   b) The number $e$, harmonic sum.
   c) Be able to use induction, binomial formula etc. to show boundedness.

2. Estimations and approximations.
   a) Triangle inequality, absolute value function.
   b) Theorem 2.5: near any number there are many rational and irrational numbers. Be able to prove and use this theorem in counterexamples (e.g. continuity salt-and-pepper function).

3. Limit of a sequence.
   a) Definition of the limit.
   b) Limit proofs: the approximation statement, how $N$ depends on $\varepsilon$ explicitly.
   c) Be able to write proper limit proofs.

4. Error term Analysis.
   a) Error term, limit proofs using error term.
   b) Newton's method.
   c) Be able to apply the definition of the limit of sequences or partial sums of infinite series, be able to find the error and show that it converges to zero.

5. The Limit Theorems.
   a) All statements. Proofs of the squeeze and limit location theorems.
   b) Subsequences. Subsequence theorem. Review here the sequential continuity theorem for functions (Chap.11).
   c) Be able to apply the theorems to prove convergence and come up with counterexamples using subsequences.

6. The Completeness Property.
   Very important chapter. When you review it, compare it with Chap.13.
   a) Nested intervals theorem. Proof.
   b) Cluster point theorem. Proof.
   c) Bolzano-Weierstrass theorem. Learn it in detail.
   d) Cauchy sequences and Cauchy criterion. Proof. Be able to apply the definition of a Cauchy sequence for partial sums of infinite series, be able to verify the Cauchy criterion.
   e) Completeness Property for sets. Investigate completeness property for compact sets.
   f) Be able to come up with counterexamples where an assumption of a theorem does not hold and the conclusion also does not hold.

7. Infinite series (7.1-6).
   a) Connection with sequences: partial sums.
   b) Basic convergence tests: 7.2-3.
   c) Completeness property for series: Cauchy criterion, monotone series, comparison arguments.
   c) Further convergence tests with applications to power series: 7.4-6.
   d) Be able to apply the tests to specific series.
   e) Be able to give examples of series that do not satisfy an assumption of a convergence test and do not satisfy the test's conclusion.

8. Power series (8.1, 8.3-4).
   a) The radius of convergence theorem.
   b) Operations with power series.
   c) Be able to apply the convergence tests from the previous chapter to specific series. Be
able to determine the radius of convergence, be able to investigate the convergence at endpoints.

9. Functions of one variable.
Definitions of: function, domain, image (p.189), composition, monotone functions, inverse functions etc.

10. Local and global behavior.
a) $\delta$-neighborhood. Review here how it is used in continuity (Chap.11), differentiability (Chap.14).
b) Completeness property. Difference between supremum and maximum (infimum and minimum).
c) Be able to determine and contrast local and global properties of functions: monotonicity, boundedness, positivity, etc.

11. Continuity and limits.
a) Definition. Be able to apply it to specific examples.
b) Discontinuity. Be able to come up with examples of different discontinuities.
c) Limits of functions and limit theorems. Know the theorems, be able to prove them and use them.
d) Sequential continuity theorem. Learn it in detail.

12. The intermediate value theorem.
a) Statements and proofs of the Bolzano’s, intermediate value, inverse function theorems.
b) Be able to come up with examples and counterexamples where an assumption of a theorem do not hold and they do not satisfy the theorem’s conclusion.
c) Zeroes. Be able to apply the intermediate value theorem when computing the number of zeroes or their location.

13. Continuous functions on compact intervals.
Very important chapter. When you review it, compare it with Chap.6.
a) Compact set. Completeness property, i.e. the sequential compactness theorem. Proof.
b) Four main theorems. Be able to state and prove them. Be able to use arguments from these theorems in exercises. Review carefully questions, exercises and problems in this chapter.

14-15. Differentiation (all Chap 14, 15.1-2)
a) Definition. Be able to apply it to specific examples.
b) Differentiation formulas and local properties. Be able to apply them.
c) Mean value theorem. Proof.
d) Linearization approximation.

18-19. Integral (all Chap 18, 19.1-2).
a) Partition. Upper, lower sums.
b) Integrability. Be able to apply the definition to specific examples.
c) Be able come up with counterexamples of non-integrable functions.
d) Statements and proofs of integrability of monotone and continuous functions. Be able to apply similar arguments to, say, continuous functions with finitely many discontinuities.
e) Statement of the Riemann integral theorem.
f) Be able to evaluate upper and lower sums directly for a given function and partition.