Memorizing basic units of information I once resisted learning English, and in particular the memorizing part. My argument is that I can always use the dictionary. Then I got a teaching assistantship for graduate study at U.C. Berkeley, and I believed I was right. The moment I stood in front of the class, I realized that I could not use my dictionary. I was able to get by for 45 minutes by talking, but then came the questions time. I could not understand them. And I could not use the dictionary.

The moral of the story: Mathematics is the language of science. Memorizing basic units of it empowers you. Understanding, memorizing, and using it comprise the phrase “master it”.

List of basic units of information in vector and tensor analysis.

Definition of tensor, scalar, vector, Kronecker delta, alternating tensor, basic examples, the stress tensor, higher-order tensors; Cartesian tensor algebra: addition of tensors, multiplication of tensors, contraction of tensors, symmetry properties of tensors.

Orthogonal coordinate transformations, polar, cylindrical, and spherical coordinate systems, the metric tensor for them, the grad div curl formula in an orthogonal curvilinear coordinate system.

Projection of a vector onto an axis; vector product, product of three vectors;

Line integrals, the theorems of Gauss, Green, and Stokes; simply and multiply connected domains; gradient, directional derivatives, divergence, curl;

Mock Exam Problems These are problems that can appear in the mid-term, but they will not anymore. The mid-term exam problems will come from the pool of homework problems. Types of problems and topics covered can be completely different.

1. Given the vectors

\[ \mathbf{A} = i_1 + 2i_2 + i_3, \quad \mathbf{B} = i_1 - i_2 + i_3 \]

where \(i_1, i_2, i_3\) form an orthonormal basis. Find
(a) The angle made by $\mathbf{A}$ and $\mathbf{B}$;
(b) The projection of $\mathbf{A}$ onto the direction of $\mathbf{B}$;
(c) The vector product $\mathbf{A} \times \mathbf{B}$.

2. Find the total flux of the vector field $\mathbf{A} = (-x_1, -x_2, -x_3)$ out of the unit sphere:
\[
x_1^2 + x_2^2 + x_3^2 = 1.
\]

3. Expand the term (i.e., undo the summation convention) $a_{ij} b_{ik}$.

4. Given the transformation of coordinates
\[
x'_i = \alpha_{ij} x_j
\]
where
\[
(\alpha_{ij}) = \begin{pmatrix}
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\
\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{6} & \frac{\sqrt{3}}{6} \\
-\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{3}
\end{pmatrix}.
\]
If a vector $\mathbf{A}$ has the components $(1, -4, 1)$ with respect to the $x_i$-coordinate system, find its components in the $x'_i$-system.

5. Form a scalar by contracting the tensor with the matrix
\[
\begin{pmatrix}
1 & 0 & 1 \\
3 & -1 & 3 \\
4 & 15 & 0
\end{pmatrix}.
\]

6. Given that
\[
(T_{ik}) = \begin{pmatrix}
1 & 0 & 2 \\
-3 & 6 & 3 \\
-4 & 2 & 4
\end{pmatrix}, \quad \mathbf{A} = i_1 + 2i_2 - i_3.
\]
Find the inner product $T_{ik} A_i$.

7. The stress tensor at a point has components given by
\[
(s_{ij}) = \begin{pmatrix}
2 & -1 & 2 \\
-1 & 3 & 0 \\
2 & 0 & 1
\end{pmatrix}.
\]
Find the stress vector $(\mathbf{p}_n)$ across an area normal to the unit vector
\[
\mathbf{n} = (i_1 - i_2 + i_3)/\sqrt{3}.
\]

8. Find the expression of $\nabla^2 f$ in cylindrical coordinate system for $f = r^2$. 