M598B: Homework Assignment 13 Hint No. 2

1. Use the formula in Section 6.11.2. Since your $\beta = 0$, you have all $B_{nm} = 0$. Since $\alpha = \phi_{10}$, you should be able to read off the coefficients in the expansion

$$\alpha(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{nm} \phi_{nm},$$

by comparison. The comparison should be like this: You are given a vector $V = (1, 1, 0)$, and you are told to write it as a linear combination of the three linearly independent vectors $(1, 0, 0), (1, 1, 0), (1, 1, 1)$ as

$$V = \alpha_1(1, 0, 0) + \alpha_2(1, 1, 0) + \alpha_3(1, 1, 1).$$

By comparison we see that $\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 0$. There is no need to solve a system of equations to find these $\alpha$'s.

2. Use eigenfunction expansion

$$u(t, x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} c_{nm}(t) \phi_{nm}(x, y)$$

where $\phi_{nm}(x, y)$ are the eigenfunctions for the Laplacian on the rectangle with zero boundary condition, see Sect. 6.11.2. Expand the source term in the same way

$$Q(t, x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} q_{nm}(t) \phi_{nm}(x, y).$$

Then derive an equation for $c_{nm}(t)$. Use the previous hint for solving the equations of $c_{nm}$.

3. Follow Monday’s lecture Section 6.12.3. Study the eigenvalue problem

$$(K(x)\phi')' + \lambda \phi = 0, \quad \phi(0) = \phi(L) = 0.$$ 

One has $\lambda_n$ and $\phi_n$. Use eigenfunction expansion

$$u(t, x) = \sum_{n=1}^{\infty} c_n(t) \phi_n(x),$$

and

$$Q(t, x) = \sum_{n=1}^{\infty} q_n(t) \phi_n(x).$$

You should be able to find

$$c_n'(t) + \lambda_n c_n = q_n(t), \quad c_n(0) = 0.$$ 

Use Chapter V Section 5.1 to integrate the equation for $c_n$. 