M598B: Homework Assignment 1


1. Prove that the projection of a sum of vectors onto any axis equals the sum of the projections of the vectors onto the same axis.
2. A parallelogram has acute angle $\pi/3$ and side lengths $a = 3, b = 5$. Thinking of the corresponding sides as vectors $\mathbf{a}$ and $\mathbf{b}$ find
   (a) The vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ (what is their geometric meaning?); (No coordinate system needed.)
   (b) The area of the parallelogram.
3. Given the vectors
   \[ \mathbf{A} = i_1 + 2i_2 + 3i_3, \quad \mathbf{B} = 4i_1 + 5i_2 + 6i_3, \]
   \[ \mathbf{C} = 3i_1 + 2i_2 + i_3, \quad \mathbf{D} = 6i_1 + 5i_2 + 4i_3; \]
   where $i_1, i_2, i_3$ are an orthonormal basis. Find
   (a) $\mathbf{A} + \mathbf{B} - \mathbf{C}$;
   (b) $\mathbf{A} \cdot \mathbf{B}$;
   (c) The angle made by $\mathbf{C}$ and $\mathbf{D}$;
   (d) The projection of $\mathbf{A}$ onto the direction of $\mathbf{B}$;
   (e) The vector product $\mathbf{A} \times \mathbf{B}$.
4. Show that the four vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are linearly dependent.
5. Verify the following identity:
   \[ \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = 0. \]