M597K: Homework Assignment 2


1. Find the derivative of the vector \( \mathbf{A}(t) = (\cos t, \sin t, 2t) \). Draw the graph of \( \mathbf{A}(t) \) with \( \mathbf{A}'(t) \). (Hand drawing is ok.)

2. Integrate the vector \( \mathbf{B}(t) = (e^t, \sin t, 2t) \) to find \( \int_0^1 \mathbf{B}(t) \, dt \). (All numbers are in radian, not degree.)

3. Evaluate the line integral
   \[ \int_L \mathbf{C} \cdot d\mathbf{r} \]
   where \( \mathbf{C} = (x_2, -x_1, -1) \) and \( L \) is a directed curve given by the graph of the vector \( \mathbf{A}(t) \) in Exercise 1 from \( t = 0 \) to \( t = 2\pi \).

4. Find the total circulation
   \[ \oint_C (x_1 + x_2)dx_1 + (x_1 - x_2)dx_2 \]
   where \( C \) is the ellipse \( \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1 \) going counter-clockwise.

5. Find the gradient of \( f = x_1^2 + x_2^2 + x_3^2 \).

6. Find a unit normal vector to the surface
   \[ x_3 = 2 - x_1 - x_2^2. \]

7. Find the divergence and curl of the vector field
   \[ \mathbf{A} = (x_2x_3, x_1x_3, x_1x_2). \]

8. Find the total flux of the vector field \( \mathbf{A} = (x_1, x_2, x_3) \) out of the unit sphere:
   \[ x_1^2 + x_2^2 + x_3^2 = 1. \]

9. Evaluate the line integral by using Green’s theorem:
   \[ \oint_C (x^2 + y^2)dx + 2xydy, \]
   where \( C \) is the square bounded by the lines \( x = 0, x = 2, y = 0, y = 2 \).

10. Let
    \[ \mathbf{F}(x_1, x_2, x_3) = (x_1\mathbf{i}_1 + x_2\mathbf{i}_2 + x_3\mathbf{i}_3)/r^3 \]
    where \( r^2 = x_1^2 + x_2^2 + x_3^2 \). Show that the flux of this vector through any closed surface \( S \) is 0 if the origin is not enclosed by \( S \).