1. (c)  
2. (b)  
3. (a)  
4. (a)  
5. (d)  
6. (d)  
7. (a) E  
   (b) F  
   (c) F  
   (d) G  
8. (a) (unstable) saddle point  
    (b) unstable improper node  
    (c) asymptotically stable spiral node  
    (d) (stable) center  
    (e) asymptotically stable improper node  
9. (a) 
   \[
   \frac{6}{s^3 + 9s} = \frac{2}{s} - \frac{2}{3} \to \frac{2}{3} - \frac{2}{3} \cos 3t
   \]
   This problem is mostly about the expansion, so, failure to expand properly must be penalized. If the squared linear factor is does not produce two terms (-4), if the coefficients are incorrect (-3) and if the Laplace lookup is incorrect (-2). Maximum penalty is 7 points.

(b) 
   \[
   e^{-s} \frac{2s - 4}{s^2 + 2s + 10} = e^{-s} \left\{ \frac{2(s + 1)}{(s + 1)^2 + 9} + \frac{2(-3)}{(s + 1)^2 + 9} \right\}
   \]
   \[
   \Rightarrow 2u_1(t)e^{-(t-1)}(\cos 3(t - 1) - \sin 3(t - 1))
   \]
   There are two important points in this problem. For the $e^{-s}$ aspect, if $u_1(t)$ does not appear but $(t - 1)$ is properly substituted (-2). If $(t-1)$ is not properly substituted (-4). Failure to appropriately determine the numerators for table lookup (resulting in table lookup error) (-4) Table lookup error (-1). Maximum penalty is 7.
10. • Rewrite the equation in the Laplace domain (4 points)

\[(s^2Y(s) - sy(0) - y'(0)) + 2(sY(s) - y(0)) + Y(s) = e^{-5t}\frac{2}{s} - e^{-10t}\]

which yields

\[Y(s) = e^{-5t}\frac{2}{s} - e^{-10t} - 4\]

\[\frac{2}{s(s+1)^2} = \frac{2}{s} - \frac{2}{s+1} - \frac{2}{(s+1)^2}\]

• Only one partial fraction expansion is necessary 2 points

\[e^{-5t}\left(\frac{2}{s} - \frac{2}{s+1} - \frac{2}{(s+1)^2}\right) \Rightarrow u_5(t) \left\{2 - 2e^{-(t-5)} - 2(t-5)e^{-(t-5)}\right\}\]

\[e^{-10s}\cdot\frac{-1}{(s+1)^2} \Rightarrow -u_{10}(t) \left\{(t-10)e^{-(t-10)}\right\}\]

\[-4\frac{1}{s^2+1} \Rightarrow -4te^{-t}\]

• So

\[y(t) = u_5(t) \left\{2 - 2e^{-(t-5)} - 2(t-5)e^{-(t-5)}\right\} - u_{10}(t) \left\{(t-10)e^{-(t-10)}\right\} - 4te^{-t}\]

11. (a) • (3 points) First, determine that the eigenvalues are -1 and -4 from

\[
\begin{vmatrix}
-2 - r & 2 \\
1 & -3 - r
\end{vmatrix} = r^2 + 5r + 4 = (r + 1)(r + 4) = 0
\]

• (3 points) The eigenvector associated with the first eigenvalue (-1) is some multiple of \(\xi_1\) where

\[\xi_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow (A - r_1I)\xi_1 = (A - I)\xi_1 = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}\xi_1\]
and the eigenvector associated with the second eigenvalue (−4) is some multiple of \( \xi_2 \) where

\[
\xi_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow (A - r_2 I)\xi_1 = (A - 4I)\xi_1 = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \xi_2
\]

- (2 points) Inserting the initial condition \( x(0) = \begin{pmatrix} 0 \\ -10 \end{pmatrix} \) yields

\[
x(t) = -\frac{10}{3} \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + \frac{20}{3} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-4t}
\]

(b) Asymptotically Stable Node

12. (a) (2,2)

(b) Beginning with the Jacobian, the linearized system at the equilibrium point \((x_o, y_o) = (2, 2)\) is

\[
\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} 2x_o - y_o & -x_o \\ y_o - 3 & x_o \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}
\]

The eigenvalues for this system are \( r = 1 \pm \sqrt{2} \) from

\[
|A - rI| = \begin{vmatrix} 1 - r & -1 \\ -2 & 1 - r \end{vmatrix} = r^2 - 2r - 1 = (r - 1)^2 - 2
\]

Since these are real roots with opposite signs, the point (2,2) is an (unstable) saddle point.

*Error in generating the Jacobian (-5) unless the Jacobian is correct but the substitution of (2,2) is incorrect (-1). Error computing the eigenvalues (-2). Error in identifying the saddle point (-2) or whatever point that is consistent with the student’s eigenvalues.*