This exam has 10 questions for a total of 100 points. **In order to obtain full credit for partial credit problems, all work must be shown.** Credit will not be given for an answer not supported by work. For other problems, points might be deducted, at the sole discretion of the instructor, for an answer not supported by a reasonable amount of work. The point value for each question is in parentheses to the right of the question number.

**You may not use a calculator on this exam. Please turn off and put away your cell phone.**

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1. (5 points) Transform the equation below into a system of first order equations.

\[ 2y^{(4)} + t^2 y''' - 2y' + 6ty = 3\sin t \]

2. (5 points) Let \( g(t) = t^2 u_1(t) + u_e(t) - (t - 1)u_\pi(t) + (t - 2)^3 u_5(t) \). Find \( g(3) \).
   (Note: \( e = 2.7 \ldots \) and \( \pi = 3.1 \ldots \))
3. (12 points) Compute the Laplace transform below. Indicate the identity you are using!

\[
\mathcal{L}\left\{u_1(t)(t - 1)e^{3(t-1)}\sin(t - 1)\right\}(s) =
\]
4. (10 points) Compute the inverse Laplace transform. Indicate the identity you are using!

\[ \mathcal{L}^{-1}\left\{ \frac{e^{-3s}}{s^3 + s^2} \right\}(t) = \]
5. (10 points) Rewrite the following piecewise continuous function $f(t)$ in terms of the unit-step function. Then find its Laplace transform.

$$f(t) = \begin{cases} 
2t^2 + t, & 0 \leq t < 3 \\
e^{4t}, & 3 \leq t
\end{cases}.$$
6. (18 points) Laplace transform can be used to solve some initial value problems by first solving for $\mathcal{L}\{y(t)\}(s)$ and then taking inverse Laplace transform to find $y(t)$. Perform the first step for problem in part (a) and the second step for problem in part (b).

(a) (6 points) For the following initial value problem:

$$y'' + 4y' + 13y = u_3(t) - \delta(t - 10), \quad y(0) = 1, \quad y'(0) = 2$$

find $\mathcal{L}\{y(t)\}(s)$. 
(b) (12 points) In the process of solving an initial value problem you found

\[ \mathcal{L}\{y(t)\}(s) = \frac{2s - 5}{s^2 + 4s + 13} - \frac{e^{-10s}}{s(s^2 + 4s + 13)}. \]

Find the solution \( y(t) \) to that initial value problem.

NOTE: The initial value problem in this part is different from that in part (a).
7. (10 points) Consider the initial value problem

\[ \vec{x}'(t) = \begin{bmatrix} -4 & 3 \\ 2 & 1 \end{bmatrix} \vec{x}(t), \quad \vec{x}(0) = \begin{bmatrix} -1 \\ 5 \end{bmatrix}. \]

(a) (6 points) Solve the initial value problem

(b) (2 points) Given that \( \vec{x}(0) = \begin{bmatrix} \alpha \\ 0 \end{bmatrix} \), and \( \lim_{t \to \infty} \vec{x}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \), find the value of \( \alpha \).

(c) (2 points) Classify the type and stability of the critical point at \((0, 0)\).
8. (10 points) Consider the initial value problem

\[ \vec{x}'(t) = \begin{bmatrix} -3 & 6 \\ -3 & 3 \end{bmatrix} \vec{x}(t), \quad \vec{x}(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}. \]

(a) (8 points) Solve the initial value problem

(b) (2 points) Classify the type and stability of the critical point at \((0, 0)\).
9. (10 points) Consider the linear system

\[ \mathbf{x}'(t) = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \mathbf{x}(t). \]

(8 points) Find the general solution of the system.

(b) (2 points) Classify the origin according to the type and stability.
10. (10 points) Consider the nonlinear system:
\[
\begin{aligned}
x' &= x(1 - x + y) \\
y' &= y(x + y)
\end{aligned}
\]

(a) (2 points) The system has 3 critical points. One of the critical points is (1,0). Find the remaining 2 critical points.

(b) (5 points) Linearize the system about the point (1,0). Clearly identify the matrix for the linearized system.

(c) (3 points) Classify the type and stability of the critical point (1,0) by examining the linearized system obtained in part (b).