This exam has 10 questions for a total of 100 points. Show all your work! In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work. For other problems, points might be deducted, at the sole discretion of the instructor, for an answer not supported by a reasonable amount of work. The point value for each question is in parentheses to the right of the question number.

You may not use a calculator on this exam. Please turn off and put away your cell phone.
1. (8 points) For each problem below, determine the order of the given differential equation; also state whether the equation is linear or nonlinear. If the equation is nonlinear, please circle the term(s) that make it so.

<table>
<thead>
<tr>
<th>Differential equations</th>
<th>Order</th>
<th>Linear/Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^2 y'' + 3y' + \sin(t)y = e^{-2t} \ln t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y^4 + (y''')^3 = \cos 5t$</td>
<td></td>
<td></td>
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<tr>
<td>$\frac{d^3 y}{dt^3} = tyy''$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y''' + ty' + (\cos^2 y)t = t^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\left(\frac{d^3 y}{dt^3}\right)^2 = \frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (6 points) Give an example of the following:

(a) (2 points) A third order partial differential equation.

(b) (2 points) A second order, linear, nonhomogeneous, ordinary differential equation.

(c) (2 points) A first order, autonomous, ordinary differential equation.
3. (10 points) Solve explicitly the initial value problem;

\[ x \frac{dy}{dx} = 2(y + 5), \quad y(3) = 4. \]
4. (10 points) Solve the initial value problem;

$$(\sin y + y \sin x) + (x \cos y - \cos x) \frac{dy}{dx} = 0.$$

You may leave your answer in implicit form.
5. (12 points) A tank initially contains 300 gal of pure water. A salt water mixture with a concentration of 5 pounds per gallon enters the tank at the rate of 20 gallon per minute. A well mixed solution leaves the tank at the same rate. Then the mixed solution is drained from the tank at the same rate. Let \( Q(t) \) be the quantity of salt in the tank at time \( t \). (pounds)

(a) (3 points) Set up the initial value problem for \( Q(t) \).

(b) (7 points) Find the particular solution of the above IVP.

(c) (2 points) How much salt is in the tank after 30 minutes?
6. (13 points) Consider the following differential equation

\[ y' = y^3(y + 2)^2(y - 3). \]

(a) (3 points) Find all of its equilibrium solutions.

(b) (5 points) Classify the stability of each equilibrium solution. Justify your answer.

(c) (2 points) If \( y(1) = 3 \), what is \( y(999) \)? Without solving the equation, briefly explain your conclusion.

(d) (3 points) Suppose \( y(1) = \beta \) and \( \lim_{t \to \infty} y(t) = 0 \). Find the value(s) of \( \beta \).
7. (8 points) Find the general solution of the following equations. Express your answer in terms of real valued functions.

(a) (4 points) \( y'' - 10y' + 25y = 0. \)

(b) (4 points) \( y'' + 4y' + 6y = 0. \)
8. (10 points) Given that \( y_1(t) = t \) is a solution to the equation,

\[
t^2y'' + 2ty' - 2y = 0, \quad t > 0.
\]

Use the method of reduction of order to find another solution \( y_2 \) which is not a scalar multiple of \( y_1 \).
9. (11 points) Consider the equation

\[ y'' - y' - 6y = 10e^{3t}. \]

(a) (3 points) Find \( y_c(t) \), the solution of its corresponding homogeneous equation.

(b) (8 points) Find the general solution of the equation.
10. (12 points) True or false: (Don’t forget to justify your answer.)
   (a) (3 points) The Existence and Uniqueness Theorem guarantees that the solution to

   \[(t^2 - 9)y' + 2 \sin(3t)y = -\frac{e^t}{t - 1}, \quad y(0) = 1.\]

   is valid on \((-3, 1)\)

   (b) (3 points) \(e^{-1+3t}, \quad 2e^{3t}\) can\textbf{not} be a fundamental set of solutions.

   (c) (3 points) The equation \(2y'' + 32y = 4 \sin 3t\) describes a mass-spring system that is un-
   dergoing resonance.

   (d) (3 points) A mass of 4 kg stretches a spring 5 meters to reach its equilibrium position. Assume \(g = 10 \text{ m/s}^2\) to be the gravitational constant. Then the spring constant is 8.