Name: _____________________________________________

SID: ____________________

Section: ____________________

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**Instructions:**

- To receive full credit, you must solve each problem on this exam fully and correctly.

- Be sure that your answers are legible and complete.

- Some questions have more than one part. Check carefully to ensure you don’t miss any parts.

- Do not write on the line marked **Score** on the bottom of each page.

- You MAY NOT use a calculator during this examination.

- There are 8 questions for a total of 100 points.
(10 points) 1. For each of the equations below state its order and whether it is linear or nonlinear.

<table>
<thead>
<tr>
<th>equation</th>
<th>order</th>
<th>linear/nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y''' + 2y' - 8y^2 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y' = y^2 - 5y + 8 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e^{2t}y'' - y' + \sin ty = \cos 2t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y''y + y' = t^3 + t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y'' + ty' + t^2y = 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(5 points) 2. The existence and uniqueness theorem guarantees that the solution to

\[ t^3 y' - \frac{3}{t-5} y = \sin t, \quad y(2) = 10 \]

uniquely exists on

1. \([-\infty, 5]\]
2. \([5, +\infty]\]
3. \([0, 5]\]
4. \([0, 10]\)
3. (a) Find and classify the equilibrium solutions of \( y' = y^2(9 - y^2) \). Justify your answer.

(b) If for some solution \( y(t) \) we know that \( y(0) = 3 \), then what is \( y(5) \)?
4. (a) Solve the initial value problem
\[ \begin{align*}
    ty' + 5y &= 7t^2 \\
    y(1) &= 0
\end{align*} \]

(b) Determine the largest interval on which the solution above exists. (Please give the reason)
(15 points) 5. Solve the initial value problem

\[ \begin{align*}
    y' &= \frac{4x-3}{2y+6} \\
    y(1) &= -5
\end{align*} \]

Write your solution in explicit form.
(10 points) 6. Find the general solution of $(3x^2y^2 + 1)dx + (2x^3y + 1)dy = 0.$
7. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gal of water. Water containing 2 lb/gal of salt enters the tank at a rate of 4gal/min. The well-stirred mixture flows out of the tank at a rate of 3 gal/min.

(a) Set up the initial value problem that models this process

(b) Solve this initial value problem

(c) How much salt does the tank contain when it begins to overflow?
8. For each of the following equations, if it is exact write the answer "exact"; if it is not exact but there exists an integrating factor write down this integrating factor.

(a) \((y \cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0\)

(b) \((3x^2 + 2xy + y^3)dx + (x^2 + y^2)dy = 0\)

(c) \(dx + (x/y - \sin y)dy = 0\)