There are 10 multiple choice questions, 8 True/False questions, and 3 partial credit questions.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

There are 14 problems on 10 pages, including this one. Check your booklet now.

The box below is for the instructor’s use.

MC .................. (50)
T/F .................... (16)
12 .................... (12)
13 .................... (10)
14 .................... (12)
Total ............... (_____)

February 18, 2003
1. (5 pts.) The equation $x^2 - 10x + y^2 + 4y = -20$ is

   a) A parabola with vertex at (5, 20)
   b) A circle with center at (5, -2)
   c) A circle with center at (-5, 4)
   d) A circle with center at (5, 2)
   e) A circle with center at (10, -4)

2. (5 pts.) The average rate of change of the function $f(x) = x^3 - x^2 + x + 1$ on the interval $[-2, 2]$ is

   a) 5
   b) -10
   c) 0
   d) $\frac{7}{2}$
   e) $-\frac{9}{2}$
3. (5 pts.) \( \lim_{x \to 2^-} \frac{x + 3}{x^2 + x - 6} = \)

a) 0  
b) \( \frac{1}{2} \)  
c) \( \infty \)  
d) \( -\infty \)  
e) The limit does not exist.

4. (5 pts.) \( \lim_{x \to -1} \frac{x^2 - x - 2}{x^2 - 1} = \)

a) \( \frac{3}{2} \)  
b) \( -\frac{1}{2} \)  
c) 0  
d) \( \infty \)  
e) The limit does not exist.
5. (5 pts.) \( \lim_{x \to 2} \frac{\sqrt{x^2 + 5} - 3}{x - 2} = \)

a) \(-\frac{1}{3}\)

b) 0

c) \(\frac{2}{3}\)

d) 1

e) The limit does not exist.

6. (5 pts.) \( \lim_{h \to 0} \frac{\sin(x + h) - \sin(x)}{h} = \)

a) 0

b) 1

c) \(\sin(h)\)

d) \(\cos(x)\)

e) The limit does not exist.
7. (5 pts.) Which of the following equations describes the line tangent to the graph of \( y = x^2 - x + \cos(x) \) at the point (0, 1)?

a) \( y = -x \)

b) \( y = 1 \)

c) \( y = -x + 1 \)

d) \( y = 2x - 1 \)

e) Not enough information is given to determine the tangent line.

8. (5 pts.) A ball is thrown upward from the top of a building. Its height at any time \( t \) (seconds) is given by \( S(t) = -16t^2 + 32t + 350 \) (feet). The maximum height that the ball reaches is

a) 254 feet

b) 350 feet

c) 366 feet

d) 382 feet

e) 398 feet
9. (5 pts.) If \( f(x) = \sec(x) \), then \( f'(\frac{\pi}{4}) = \)
   
   a) \(-\frac{1}{2}\)
   
   b) \(\sqrt{2}\)
   
   c) \(-\frac{\sqrt{2}}{2}\)
   
   d) 1
   
   e) 2

10. (5 pts.) \( \tan(\sin^{-1}(\frac{3}{5})) = \)
    
    a) \(\frac{3}{4}\)
    
    b) \(\frac{3}{5}\)
    
    c) \(\frac{4}{5}\)
    
    d) 1
    
    e) \(\frac{5}{3}\)
11. (16 pts., 2 pts. each) True or False:

a) T  F  The function \( f(x) = -x^3 + 2x + 1 \) has a root between \( x = 1 \) and \( x = 2 \).

b) T  F  If \( f(x) \) is continuous at \( x = c \), then \( f'(x) \) exists at \( x = c \).

c) T  F  If \( f(x) = \pi^3 + \pi x \), then \( f'(x) = 3\pi^2 + \pi \).

d) T  F  If the graph of \( y = f(x) \) has a unique tangent line at \( x = c \), then \( f(x) \) is differentiable at \( x = c \).

e) T  F  If \( f'(x) \) exists at an interior point \( x = c \), then \( \lim_{x \to c^+} f(x) = f(c) \).

f) T  F  For all angles \( x \), \( \tan^2(x) + 1 = \sec^2(x) \).

g) T  F  If \( f(x) = \frac{\sin(x)}{\cos(x)} \), then \( f'(x) = \frac{1}{\cos^2(x)} \).

h) T  F  For any two angles \( \alpha, \beta \) : \( \cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta) \).
12. (12 pts.) For each function below, find its derivative

DO NOT SIMPLIFY YOUR ANSWER!

a) \( f(x) = \frac{3x^3 - 4x^2 - x + 3}{\sin^2(x)} \)

b) \( f(x) = \cos^5(\tan(2x^2 - x)) \)
13. (10 pts.) Suppose
\[ f(x) = \begin{cases} 
\frac{x^2 - 4}{x(x-2)} & x \leq 3 \\
\frac{10}{(x+1)(x-5)^2} & x > 3 
\end{cases} \]

Find all discontinuities. For each discontinuity, determine and verify its type.
14. (12 pts.) a. State the limit of definition of the derivative.
   \[ f'(x) = \]

b. Use the limit definition to compute the derivative of
   \[ f(x) = \sqrt{ax + b} \]
   where \( a \) and \( b \) are constants.

   (No credit is given if any other method is used to find the derivative!)