1. Compute \( \Delta y \) and \( dy \) for \( y = \frac{16}{x} \), \( x = 4 \) and \( \Delta x = -1 \).

2. Find the absolute maximum and minimum of the function \( f(x) = \sin(2x) + \cos(2x) \) on the interval \([0, \frac{\pi}{4}]\).

3. Which one of the following statements is true for the function \( f(x) = \sqrt{x^2 - 9} \) on \([3,5]\)?
   (a) The Mean Value Theorem (MVT) can be applied to this function on this interval, and the only \( c \) value that satisfies the conclusion of the MVT is \( c = 2 \).
   (b) The MVT can be applied to the function on this interval, and the only \( c \) value that satisfies the conclusion of the MVT is \( c = \sqrt{12} \).
   (c) The MVT cannot be applied because \( f(3) \neq f(5) \).
   (d) The MVT cannot be applied because \( f(x) \) is not continuous for all real numbers.
   (e) The MVT cannot be applied because \( f'(3) \) does not exist.

4. Based on the function \( g(x) = x^4 - 4x^3 + 10 \)
   (a) Determine intervals of increasing and/or decreasing.
   (b) Determine any local maximum or minimum points, \((x, y)\).
   (c) Determine concavity and any points of inflection.
   (d) Sketch the graph based on the above information. Do not bother finding the x-intercepts.

5. Which of the following statements is incorrect?

   (a) The graph of \( y = \frac{x}{x^2 + 4} \) has a horizontal asymptote at \( y = 0 \).
   (b) The graph of \( y = \frac{x^2}{x + 4} \) has no horizontal asymptote.
   (c) The graph of \( y = \frac{x}{x^2 - 4} \) has two vertical asymptotes.
   (d) The graph of \( y = \frac{x}{x^2 - 4} \) has a vertical asymptote at \( x=4 \).
   (e) The graph of \( y = x^5 + 2x^3 \) is symmetric about the origin.

6. Find all of the asymptotes (vertical, horizontal, slant) for the function \( f(x) = \frac{x^3}{2x^2 - 8} \).

7. What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?
8. Evaluate \( \lim_{x \to -\infty} \frac{\sqrt{9x^2 - 2}}{6 - 5x} \). (See #23 for more problems like this)

9. Review definition 6 on page 201 and then answer the following question:
   What are the critical number(s) of the function \( f(x) = \frac{\sqrt{x - 2}}{x + 4} \)?
   (a) -4 and 2 only
   (b) 2 only
   (c) 0, 2 and 5 only
   (d) 2 and 5 only
   (e) 5 and -4 only

10. Suppose that \( f \) is continuous on \([0,4]\), \( f(0) = 1 \), and \( 2 \leq f'(x) \leq 5 \) for all \( x \) in \((0,4)\). Show that \( 9 \leq f(4) \leq 21 \).

11. Suppose \( f''(x) = (x - 1)(x - 2)^2(x - 3)^3 \). What are the x-coordinates of the inflection points of \( y = f(x) \)?
   (a) \( x = 2 \) only
   (b) \( x = 2 \) and \( x = 3 \) only
   (c) \( x = 1 \) and \( x = 3 \) only
   (d) \( x = 1, x = 2, \) and \( x = 3 \)
   (e) \( y = f(x) \) does not have any inflection points.

12. Evaluate \( \lim_{x \to 0} \frac{(2x^3 - 3x)^3}{15x - 2x^3 + x^4} \).

13. Given \( f(x) = \frac{x^3 - 1}{x^3 + 1} \), fill in each of the following:
   (a) Domain
   (b) Intercepts
   (c) Symmetry?
   (d) Asymptote(s)?
   (e) Interval(s) where the function is increasing? Decreasing?

14. You are planning to make an open rectangular box from an 8-by-15-in. piece of cardboard by cutting squares from the corners and folding up the sides. What are the dimensions of the box of largest volume you could make this way?
15. Determine symmetry, intercepts and the limits as \( \rightarrow -\infty, -\infty \) for the function \( f(x) = x(x + 2)^3 \).

16. Evaluate \( \lim_{x \to \infty} \left( \sqrt{x^2 + ax} - \sqrt{x^2 + bx} \right) \).

17. Given \( g(x) = \frac{x^2 - 3}{2x - 4} \), find any vertical, horizontal or slant asymptotes.

18. Find the linearization \( L(x) \) of \( f(x) = \sqrt[3]{x} \) at \( a = -8 \).

19. Find the number \( c \) that satisfies the conclusion of the Mean Value Theorem for \( f(x) = \frac{x}{x + 6} \) on the interval \([0,1]\).

20. Given \( f(x) = x - 2 \cos x \) on \( 0 \leq x \leq 2\pi \).

   (a) Find all critical numbers.
   (b) Determine local extrema.

21. It took 10 seconds for a thermometer to rise from -10 degrees to 100 degrees when the thermometer was removed from a freezer and placed in boiling water. Show that there was some moment of time at which the temperature was rising at exactly 11 degrees per second. (Hint: Let \( f(t) \) represent the temperature on the thermometer at time \( t \). Assume \( f(t) \) is a differentiable function and use the Mean Value Theorem to reach your conclusion.)

22. Evaluate \( \lim_{x \to \infty} \frac{(5x^3 + 9x)(2x^2 - 3x)^2}{10x^7 - 2x^5 + 15x} \).

23. Evaluate (a) \( \lim_{x \to \infty} \frac{\sqrt{9x^2 + 6}}{1 + 4x} \) (b) \( \lim_{x \to -\infty} \frac{\sqrt{9x^2 + 6}}{1 + 4x} \) (c) \( \lim_{x \to \infty} \frac{\sqrt{9x^2 + 6}}{1 - 4x} \) (d) \( \lim_{x \to -\infty} \frac{\sqrt{9x^2 + 6}}{1 + 4x} \)

General Suggestions:

- Review Differentiation formulas.
- Go back to section 1.8 and review continuity; this plays a very important role in many theorems as well as in graphing.
- Now review the following theorems (suggestion, write out the hypotheses and conclusion for each):
  - The Extreme Value Theorem
  - Fermat’s Theorem
  - Rolle’s Theorem
  - The Mean Value Theorem
  - Theorem 4 (pg 434)
SELECT ANSWERS

1. $\Delta y = \frac{4}{3}, dy = 1$;  
2. Max: $y = \sqrt{2}$;  
Min: $y = 1$

3. Choice (b);  
4. (a) Dec $(-\infty, 0)$, (0,3), Inc $(3, \infty)$, (b) Min @ (3,-17), (c) IP (0,10) and (2,-6)

5. Choice (d);  
6. Vertical: $x = 2$, $x = -2$, Horizontal: None, Slant: $y = \frac{1}{2}x$

7. $25/4$ cm$^2$;  
8. $\frac{3}{5}$

9. Choice (d);  
10. HINT: Use the MVT.

11. Choice (c);  
12. The limit is equal to 4.

13. (a) $(-\infty,-1), (-1, \infty)$;  
(b) $(0,-1), (1,0)$;  
(c) None

(d) $x = -1$;  
(e) $y = 1$;  
(f) Increasing over $(-\infty,-1), (-1, \infty)$;  
(g) None

14. $\frac{14}{3}x - \frac{35}{3}x - \frac{5}{3}$ inches

15. Symmetry: None (The easiest way to do this is to multiply it out.)

Intercepts: (0,0) and (-2,0)

Limits: As $x \to \infty$, $f(x) \to \infty$ and as $x \to -\infty$, $f(x) \to \infty$

16. The limit is equal to $\frac{a-b}{2}$

17. Vertical: $x = 2$;  Horizantial: None;  Slant: $y = \frac{1}{2}x + 1$

18. $L(x) = \frac{1}{12}x - \frac{4}{3}$

19. $c = -6 + \sqrt{42}$

20. (a) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$  
(b) Max @ $x = \frac{7\pi}{6}$, Min @ $x = \frac{11\pi}{6}$

21. Proof using MVT

22. The limit is equal to 2.

23. (a) $\frac{3}{4}$  
(b) $-\frac{3}{4}$  
(c) $-\frac{3}{4}$  
(d) $\frac{3}{4}$