1. (20 points) Short questions

(a) Determine whether the following matrix is singular or nonsingular and explain your answer:

\[
A = \begin{pmatrix}
  3 & -6 & 0 \\
  1 & 0 & 0 \\
  5 & 9 & 1
\end{pmatrix}
\]

(b) Assume that \( A \) and \( B \) are two \( 3 \times 3 \) matrices such that \( B = 2A \) and \( |A| = 2 \). Find the value of \( |B| \).

(c) “If \( x_0 \) is a critical point, then \( f(x_0) \) is either a maximal or a minimal value”. Is this statement true? Please explain.

(d) Is it always true that the average revenue of a firm is equal to its marginal revenue at a perfect competitive market? Please explain.

(e) Given the input matrix \( A \) in Leontief Input-Output model, where \( A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \). Then \( a_{11} + a_{21} \leq 1 \) or \( a_{11} + a_{12} \leq 1 \)? Please indicate your answer and explain.
2. (20 points) Suppose that the demand and supply functions are given as follows:

\[ Q_{d1} = 10 - 2P_1 + 3P_2, \quad Q_{s1} = 2 + 3P_1^2 \]
\[ Q_{d2} = 15 + P_1 - P_2, \quad Q_{s2} = -1 + 2P_2 \]

What is the equilibrium solution?
3. (10 points) Given the national income model:

\[ \begin{align*}
Y &= C + I_0 + G_0, \\
C &= a + b(Y - T), \\
T &= d + tY,
\end{align*} \]

(List the endogenous variables in the order \(Y, C, T\))

(a) Write the model in matrix form.

(b) Find the equilibrium national income.
4. (10 points) Given the total cost function \( C(Q) = a + bQ + cQ^2 \) where output \( Q > 0 \) and \( a, b, c, \) are positive constants, find the \( Q > 0 \) which minimize the average cost.
5. (20 points) A perfectly competitive producer has a production function of form $Q = 40L^{1/2}$. The output he produces is sold at a price $5 per unit. His costs of production consist of $50 fixed costs per period plus $25 per period for each unit of labor he employs.

(a) Find the profit maximizing input level of labor.

(b) What is the relation between the value of the marginal product of labor and the wage rate at the maximizing input level of labor?
6. (10 points) Given the demand function \( p = 60 - 3Q \), find the total revenue function (\( R \)), the marginal revenue (\( MR \)) and the price elasticity of demand at \( Q = 5 \).

7. (10 points) For \( y^3x^2 + yx + x^2 - 4 + y = 0 \), find the value of \( \frac{dy}{dx} \) at \( (1, 1) \).