Midterm Exam

Name

Section
1. (20 points) Short questions

(1) If $A$ is a $3 \times 3$ matrix and $B$ is a $3 \times 4$ matrix what is the dimension of $AB$ and $BA$ if it is defined.

(2) What does matrix $A$ nonsingular mean?

(3) “$A$ is a square matrix and $A^2 = 0$ then $A = 0$”, If the statement is true, give a reason. If not, give a example.

(4) The average revenue of a firm is $f(Q)$, what is it’s marginal revenue?
2. (20 points)
Given the nonlinear market model:

\[ Q_s = -4 + 2P^2 \] and \[ Q_d = 12 - 4P \]

Find the economically meaningful equilibrium solution (price and quantity).
3. (20 points) Given

\[-3x_1 + 4x_2 - 2x_3 = -10,\]
\[4x_1 + 3x_3 = 11,\]
\[-x_1 + \beta x_3 = 16,\]

1. Write the equation system in the matrix format $AX = b$.

2. Find the condition so that $AX = b$ has unique solution.

3. Find the solution $x_3$ under the condition.
4. (20 points)
A monopolist’s average revenue function is \( AR(Q) = 20 - 30Q \) and his total cost function is \( C(Q) = 20Q^2 \). Find:

(1) His total revenue function

(2) His profit-maximizing output level and maximum profit.

(3) The price per unit at which this profit-maximizing output is sold.
5. (20 points) (1) A firm has a production function \( Q = g(L) \). Demonstrate that \( AP = MP \) when \( AP \) is at a maximum.

(2) Demonstrate why the difference between marginal cost and average cost has always the same sign as the slope of the average cost curve.