Economics 480

Professor: Jenny X. Li

Final Exam

Name
1. (20 points) Short questions

(1) Is it true that Kuhn-Tucker conditions are the necessary conditions for optimization problems with inequality constraints? Explain.

(2) Find the value of \( x \) that satisfies the equation \( 2(3^x) - 5 = 0 \).

(3) Write an exponential expression for the value: $40 compounded continuously at the interest rate of 5\% for 3 years.

(4) Give the definition of the price elasticity \( E_d \) of demand for a demand function \( Q = f(p) \). Especially for \( Q = 50 - P^2 \) determine whether the demand is elastic at \( P = 5 \).
(5) Find the continuous-compounding nominal interest rate per annual (r) that is equivalent to a
discrete-compounding interest rate (i) of 4 percent per annum, compounded semianually.

(6) Suppose a certain wine dealer has a case of wine and the growing value \( V \) of the wine is know
to be the following functions of time: \( V = Ke^{\sqrt{t}} \). Let’s assume that the interest rate on the
continuous-compounding basis is at the level of \( r \). What is the present value of \( V \) at time \( t \)?
2. (12 points) In a two-industry economy, it is known that industry I uses 10 cents of its own product and 60 cents of commodity II to produce a dollar’s worth of commodity I; Industry II uses none of its own product but uses 50 cents of commodity I in producing a dollar’s worth of commodity II; and the open sector demands $125 of commodity I and $215 of commodity II.

1. Write out the input matrix, the Leontief matrix, and the specific input-output matrix equation.

2. Find the solution output level.
3. (13 points) Find the derivatives of $y$

\[ y = \frac{x^2 e^{x^2+1}}{(x + 8)(3x + 5)} \]

Find the extreme values of $z$, if any. Check whether they are maxima or minima.

\[ z = x^2 + xy + 2y^2 - 2x - y \]
4. (15 points) A monopolist sells his output in two markets. His total costs and average revenue functions are $C(Q) = 12Q + 4$, where $Q = Q_1 + Q_2$; $AR_1 = 60 - 3Q_1$; $AR_2 = 20 - 2Q_2$. Find his profit maximizing output in each market, his maximum profit, the price he will charge in each market, and the elasticity of demand in each market.
5. (12 points) The demand function for a monopolist’s two products are \( Q_1 = 40 - 2P_1 + P_2 \) and \( Q_2 = 30 + P_1 - P_2 \) respectively. His total cost function is \( C = Q_1^2 + 2Q_1Q_2 + Q_2^2 + 95 \). Find his profit-maximizing output level for each product.
6. (15 points) A consumer’s utility function is $U(x, y) = 2 \ln x + \ln y$, His equality budget constraint is $P_x = 2$, $P_y = 4$ and $B = 36$.

1. Write the Lagrangian function.

2. Find the optimal levels of purchase $x$ and $y$.

3. Is the second-order sufficient condition for maximum satisfied?
7. (13 points) Given

\[
\begin{align*}
\text{Max } & \quad x - y^2 \\
\text{subject to } & \quad -(10 - x^2 - y)^3 \leq 0 \\
& \quad -x \leq -2 \\
& \quad x, y \geq 0
\end{align*}
\]

Write out the Kuhn-Tucker conditions for the problem.
Bonus Problem (15 points)
A firm is planning to invest a theater that offer shows in the evening (peak period) and matinees (off-peak period). The peak period demand function is $P_1 = 200 - 2Q_1$ and off-peak period demand is $P_2 = 90 - Q_2$. The variable cost is 10 per unit and capacity costs 10 per unit which is only paid once. Find the optimal outputs and capacity for this problem.