On your scantron, write and bubble your PSU ID, Section Number, and Test Version.

On your scantron, bubble letters corresponding to your answers on indicated questions. It is a good idea for future review to circle your answers in the test booklet.

Check that your exam contains 20 questions numbered sequentially.

Answer Questions 1-20 on your scantron.

THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED IN THIS EXAMINATION.

THE USE OF NOTES OF ANY KIND IS NOT PERMITTED DURING THIS EXAMINATION.
1. Find the intervals where \( f(x) = x^4 - 8x^2 + 2008 \) is increasing.
   a) \((-2, 2)\).
   b) \((-2, 0)\) and \((2, \infty)\).
   c) \((-\infty, -2)\) and \((0, 2)\).
   d) \((-\infty, -2)\) and \((2, \infty)\).

2. Find the \( x \)-coordinate of each inflection point of the graph of the function \( f(x) = x^5 - 5x^4 - 18 \).
   a) \(x = 0\) and \(x = 3\)
   b) \(x = 0\)
   c) \(x = 3\)
   d) \(x = 0\) and \(x = 4\)

3. Determine all vertical and horizontal asymptotes of \( g(x) = \frac{x^2 + 3x + 2}{x^3 - x} \).
   a) vertical asymptotes at \(x = 1\) and \(x = 0\); horizontal asymptote at \(y = 0\).
   b) vertical asymptotes at \(x = 1\), \(x = 0\) and \(x = -1\); horizontal asymptote at \(y = 1\) and \(y = -2\).
   c) vertical asymptotes at \(x = 1\), \(x = 0\) and \(x = -1\); horizontal asymptote at \(y = 0\).
   d) vertical asymptotes at \(x = 1\) and \(x = 0\); no horizontal asymptote.
4. Suppose that the first derivative of \( f(x) \) is given by \( f'(x) = \frac{(x^2 - 9)(x + 1)(x - 1)^2}{x^2 + 7} \). Find all the relative minima of \( f \), if any.

   a) \( f(x) \) has relative minima at \( x = -3, x = -1, x = 1, \) and \( x = 3 \).
   
   b) \( f(x) \) has relative minima at \( x = -1 \).
   
   c) \( f(x) \) does not have any relative minima.
   
   d) \( f(x) \) has relative minima at \( x = -3 \) and \( x = 3 \).

5. Find all critical points for the function \( f(x) = x + \frac{9}{x} + 2 \).

   a) \( x = 0, x = -3 \) and \( x = 3 \)
   
   b) \( x = -3 \) and \( x = 3 \)
   
   c) \( x = 0 \)
   
   d) \( x = 0 \) and \( x = 3 \)

6. Suppose the second derivative of \( f(x) \) is given by \( f''(x) = \frac{e^x(x + 3)(x - 2)}{x + 1} \). Determine the intervals where \( f(x) \) is concave up.

   a) \((-3, 2)\)
   
   b) \((-\infty, -3) \) and \((-1, 2)\)
   
   c) \((-\infty, -3) \) and \((2, \infty)\)
   
   d) \((-3, -1) \) and \((2, \infty)\)
7. Find the relative extrema, if any, of the function \( f(x) = \frac{x}{x+1} \).

   a) relative minimum at \( x = 0 \).
   b) relative maximum at \( x = -1 \).
   c) relative minimum at \( x = -1 \).
   d) \( f(x) \) has no relative extrema.

8. If \( xy = 1 \) and \( x \) is guaranteed to be positive, what is the minimal value of \( x + y \)?

   a) \( \frac{1}{2} \)
   b) 1
   c) 2
   d) \( \frac{3}{2} \)

9. The profit function in dollars for producing \( x \) units of a certain product is \( P(x) = -2x^3 + 6x + 400 \). What is the largest profit for \( x \) between \([0, 3]\)?

   a) $400
   b) $404
   c) $410
   d) $398
10. Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 + 4x^3$ on the interval $[-2, 1]$.

   a) absolute minimum value: $-1$
       absolute maximum value: $0$

   b) absolute minimum value: $-1$
       absolute maximum value: $16$

   c) absolute minimum value: $0$
       absolute maximum value: $7$

   d) absolute minimum value: $-2$
       absolute maximum value: $-1$

11. An open box is to be made by cutting a square from each corner of a 12-inch by 12-inch piece of metal and then folding up the sides. What size square should be cut from each corner to produce a box of maximum volume?

   a) 2
   b) 3
   c) 4
   d) 1

12. Simplify $4 \log_3 81 + \ln \left( \frac{1}{e^7} \right) - 2 \log_4 1$.

   a) 5
   b) 23
   c) 9
   d) 10
13. If $5^{t^2-3} = \left( \frac{1}{25} \right)^t$, what is $t$?

   a) $t = -1, 3$
   b) $t = 1, -3$
   c) $t = -1, 2$
   d) $t = 1, 3$

14. Find the interest rate needed for an investment of $7000 to triple in 10 years if interest is compounded continuously.

   a) rate = \( \frac{1 + \left( \frac{1}{3} \right)^3}{10} \)
   b) rate = \( \frac{\ln(3)}{3} \)
   c) rate = \( \frac{\ln\left( \frac{1}{3} \right)}{10} \)
   d) rate = \( \frac{\ln(3)}{10} \)

15. If $400$ is invested at 4.8% per year compounded quarterly, what will be the accumulated amount after 5 years?

   a) $400(1.48)^5$
   b) $400(1.12)^5$
   c) $400(1.012)^{20}$
   d) $400(1.48)^{20}$
16. If the half-life of a radioactive substance is 1590 years and the initial amount of the substance present is 100 mg, find a formula for the mass, \( Q(t) \), that remains after \( t \) years.

   a) \( Q(t) = 100e^{\ln(2)t} \)

   b) \( Q(t) = 100e^{\frac{\ln(1/2)}{1590}t} \)

   c) \( Q(t) = 100e^{1590\ln(1/2)t} \)

   d) \( Q(t) = e^{\ln(1/2)t} \)

17. Compute the derivative of \( y = -3e^{3x^2+5} \).

   a) \( y' = -18xe^{3x^2+5} \)

   b) \( y' = -3(3x^2 + 5)e^{3x^2+5} \)

   c) \( y' = -9e^{3x^2+5} \)

   d) \( y' = -e^{3x^2+5} + (-9x^2 + 15) \)

18. Compute the derivative of \( y = \frac{e^x}{2x + 1} \).

   a) \( y' = \frac{e^x - 1}{(2x + 1)^2} \)

   b) \( y' = \frac{e^x}{2x + 1} \)

   c) \( y' = \frac{e^x(2x + 3)}{(2x + 1)^2} \)

   d) \( y' = \frac{e^x(2x - 1)}{(2x + 1)^2} \)
19. Compute the derivative of \( f(x) = 3x \ln x^2 \).

   a) \( f'(x) = \frac{3}{x^2} \)

   b) \( f'(x) = 6x + 3 \ln x^2 \)

   c) \( f'(x) = 6 + 3 \ln x^2 \)

   d) \( f'(x) = \frac{6}{x} + 3 \ln x^2 \)

20. Find the equation of the line tangent to \( f(x) = (x^2 + 1)e^{3x} \) at the point \((0,1)\).

   a) \( y = x + 1 \).

   b) \( y = xe^{3x}(3x^2 + 2x + 1) + 1 \).

   c) \( y = 3x + 1 \).

   d) \( y = 1 \).