You may be tempted to say 1,000 and 780, respectively, but things are not quite as simple as that. We have to take into account the interindustry demand as well. Let us say that industry C produces electricity. Of course, producing almost any product will require electric power. Suppose that industry B needs $10 \times 10^6$ worth of electricity for each $1$ of output C produces and that industry A needs $20 \times 10^6$ worth of B’s products for each $1$ of output A produces. Find the outputs a and b needed to satisfy both consumer and interindustry demand.

21. Find the outputs a and b needed to satisfy the consumer and interindustry demands given in the following figure (see Exercise 20):

22. Consider the differential equation

\[
\frac{d^2x}{dt^2} - \frac{dx}{dt} - x = \cos(t).
\]

This equation could describe a forced damped oscillator, as we will see in Chapter 9. We are told that the differential equation has a solution of the form

\[x(t) = a \sin(t) + b \cos(t).\]

Find a and b, and graph the solution.

23. Find all solutions of the system

\[
\begin{align*}
7x - y &= \lambda x, \\
-6x + 8y &= \lambda y,
\end{align*}
\]

a. \( \lambda = 5 \)  

b. \( \lambda = 10 \)  

c. \( \lambda = 15 \).

24. On your next trip to Switzerland, you should take the scenic boat ride from Rheinfall to Rheinau and back. The trip downstream from Rheinfall to Rheinau takes 20 minutes, and the return trip takes 40 minutes; the distance between Rheinfall and Rheinau along the river is 8 kilometers. How fast does the boat travel (relative to the water), and how fast does the river Rhein flow in this area? You may assume that both speeds are constant throughout the journey.

25. Consider the linear system

\[
\begin{align*}
x + y - z &= -2, \\
x - y + 2z &= 5, \\
3x - 5y + 13z &= 18,
\end{align*}
\]

where \( k \) is an arbitrary number.

a. For which value(s) of \( k \) does this system have one or infinitely many solutions?

b. For each value of \( k \) you found in part a, how many solutions does the system have?

c. Find all solutions for each value of \( k \).

26. Consider the linear system

\[
\begin{align*}
x + y - z &= 2, \\
x + 2y + z &= 3, \\
x + y + (k^2 - 5)z &= k
\end{align*}
\]

where \( k \) is an arbitrary constant. For which value(s) of \( k \) does this system have a unique solution? For which value(s) of \( k \) does the system have infinitely many solutions? For which value(s) of \( k \) is the system inconsistent?

27. Emile and Gertrude are brother and sister. Emile has twice as many sisters as brothers, and Gertrude has just as many brothers as sisters. How many children are there in this family?

28. In a grid of wires, the temperature at exterior mesh points is maintained at constant values (in °C) as shown in the accompanying figure. When the grid is in thermal equilibrium, the temperature \( T \) at each interior mesh point is the average of the temperatures at the four adjacent points. For example,

\[
T_2 = \frac{T_3 + T_1 + 200 + 0}{4}.
\]

Find the temperatures \( T_1, T_2, \) and \( T_3 \) when the grid is in thermal equilibrium.

29. Find the polynomial of degree 2 (a polynomial of the form \( f(t) = a + bt + ct^2 \) whose graph goes through the points \((1, -1), (2, 3),\) and \((3, 13)\). Sketch the graph of this polynomial.