Problem 1: Use the comparison test to show that the following series converge:
(a) \[ \sum_{n} \frac{\sqrt{n+1} - \sqrt{n}}{n} \]
(b) \[ \sum_{n} \frac{1}{n^2 - \ln n} \]
You may use \( \ln x < x \) for \( x > 0 \).

Problem 2: Let \( f, g : \mathbb{R} \to \mathbb{R} \) be functions. Prove, using the sequence definition, that if \( f, g \) are continuous at \( x \), then

1. \( h := f + g \) is continuous at \( x \).
2. \( h := f - g \) is continuous at \( x \).
3. \( h := fg \) is continuous at \( x \).

Problem 3: Let \( f : \mathbb{R} \to \mathbb{R} \) be a function. Prove, using the \( \epsilon-\delta \) definition of continuity that if \( f(x) \neq 0 \) then there is some interval of the form \( (x-\delta, x+\delta) \) where \( f \) is non-zero.

Problem 4: Let \( f, g : \mathbb{R} \to \mathbb{R} \) be functions. Use the previous exercise to show that if \( f, g \) are continuous at \( x \), and \( g(x) \neq 0 \) then \( h(y) := f(y)/g(y) \) is well-defined in some interval of the form \( (x-\delta, x+\delta) \) and \( h \) is continuous at \( x \).

Problem 5: Prove that \( f(x) = \cos x \) is continuous at every \( x \in \mathbb{R} \).

Problem 6: Let \( f, g \) be two continuous functions on \([a, b]\), and assume that \( f(a) < g(a) \), but \( f(b) > g(b) \). Prove that \( f(x) = g(x) \) for some \( a < x < b \).

Problem 7: Prove that every function \( f : \mathbb{R} \to \mathbb{R}, f(x) = x^n \) with \( n \in \mathbb{N} \), is continuous at every \( x \in \mathbb{R} \).