As in class, we start from the Divergence Theorem for a vector field $\vec{F}$.

**Divergence Theorem:** Let $F$ be a smooth vector field defined in $\Omega$. Then
\[
\int_{\Omega} \nabla \cdot \vec{F} \, dV = \int_{\partial \Omega} \vec{F} \cdot n \, dS
\]
where $n$ is the outward normal to $\partial \Omega$.

We now consider two arbitrary but smooth (at least $C^2$) scalar fields, $f : \Omega \to \mathbb{R}$ and $g : \Omega \to \mathbb{R}$. We then take the vector field $\vec{F} = f \nabla g$. Applying the Divergence Theorem to this field:
\[
\int_{\Omega} \nabla \cdot (f \nabla g) \, dV = \int_{\partial \Omega} (f \nabla g) \cdot n \, dS
\]
and using the product rule on the LHS leads to one of the Green’s Theorems, or Identities:

**Green’s Identity I:**
\[
\int_{\Omega} f \Delta g \, dV = -\int_{\Omega} \nabla f \cdot \nabla g \, dV + \int_{\partial \Omega} f (n \cdot \nabla g) \, dS
\]
where $\Delta g = \nabla^2 g$ is the Laplacian of $g$, and again $n$ is the outward normal to $\partial \Omega$.

We now repeat this process with $\vec{F} = g \nabla f$, which leads to
\[
\int_{\Omega} g \Delta f \, dV = -\int_{\Omega} \nabla g \cdot \nabla f \, dV + \int_{\partial \Omega} g (n \cdot \nabla f) \, dS
\]
and subtract this from the result obtained with $\vec{F} = f \nabla g$. This leads to

**Green’s Identity II:**
\[
\int_{\Omega} (f \Delta g - g \Delta f) \, dV = \int_{\partial \Omega} (f (n \cdot \nabla g) - g (n \cdot \nabla f)) \, dS.
\]