1. Restate the Sampling Theorem (given in class). Consider a series of data points \( A(t_j) \) with \( t_j = t_0 + j\Delta t \), for integer \( j \), \( t_0 \) and \( \Delta t \) real positive constants.

(a) what would the Nyquist frequency be if it is known that \( A(t_j) \) accurately captures the underlying function \( A(t) \)?

(b) Given a continuous signal in which you are attempting to observe an oscillation at a frequency of 510 Hz, what minimum \( \Delta t \) would be required to avoid aliasing?

2. Keener 2.2.8 (a, b, d) (p.96)

3. The heat equation for the temperature \( T = u(r, \theta, t) \) of a disk of radius \( R \) obeys the partial differential equation
\[
 u_t = k\nabla^2 u, \quad r < R, \; t > 0
\]
(where \( \nabla^2 \) is the Laplacian operator). Show that for a circularly symmetric temperature distribution (\( \partial u / \partial \theta = 0 \)), the spatial part of the separation of variables solution \( u(r, t) = f(r)g(t) \) obeys a Sturm-Liouville problem.

4. The Haar wavelets form a complete orthonormal set on \((−\infty, \infty)\). We can define them using a function \( \phi \) such that \( \phi(x) = 1 \) for \( x \in [0, 1) \), and \( \phi = 0 \) otherwise.

Define \( \psi(x) \equiv \phi(2x) - \phi(2x - 1) \); then the Haar wavelets are
\[
 \Psi_{mn}(x) = 2^{m/2}\psi(2^m x - n)
\]
for \( m, n \) any positive or negative integer (see also Keener pp.79-88).

(a) First sketch \( \psi(x) \), then \( \Psi_{mn} \) for \( m, n = 0, \pm 1, \pm 2 \).

(b) If \( f(x) \) is in \( L^2(−\infty, \infty) \), and is decomposed
\[
 f(x) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} c_{mn} \Psi_{mn},
\]
find a simple formula for the coefficients \( c_{mn} \).

5. Keener 3.2.1 (p.128)