MATH 580: Problem Set 3
due Thursday, October 2, 2003

1. For each \( A \), specify for what \( b \) the equation \( Ax = b \) has a solution:

\[
A = \begin{pmatrix} 2 & 3 \\ 14 & 21 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 13 \\ -8 & 0 \end{pmatrix}; \quad D = \begin{pmatrix} 0 & \varepsilon & \varepsilon \\ \varepsilon & 0 & \varepsilon \\ \varepsilon & \varepsilon & 2\varepsilon \end{pmatrix},
\]

with \( \varepsilon \in \mathbb{R} \) (as usual, use the Euclidean inner product).

2. For the real symmetric matrix

\[
B = \begin{pmatrix} a & b \\ b & c \end{pmatrix},
\]

show that the eigenvalues \( \lambda_i \) are both real.

3. Consider the matrix

\[
A = \begin{pmatrix} 4 & 8 \\ 1 & 2 \end{pmatrix}
\]

(a) find the eigenvalues of \( A \), and the transformation matrix \( T \) which diagonalizes \( A \);

(b) find the eigenvalues of the matrix \( B = AA \equiv A^2 \);

(c) in general, if \( A \) is a \( n \times n \) matrix with eigenvalues \( \{ \lambda_i \} \), what are the eigenvalues of \( A^2 \)?

4. Show that if the \( n \times n \) matrix \( A \) has the eigenvalue \( \lambda_* \) for the eigenvector \( x_* \), that for any \( \beta \in \mathbb{C} \), the matrix \( B = A + \beta I \) has the eigenvalue \( \lambda_* + \beta \) with the same eigenvector.

5. Keener 1.2.3 (p.51)

6. Keener 1.2.6 - b,c (p.51)

7. Keener 1.5.1 - a,b,c (p.54)

8. Keener 1.5.3 (p.55)