MATH 580: Problem Set 10
due Tuesday, November 25, 2003

1. Aris 2.35.1 (p.19)

2. Let $\mathbf{A} = \vec{u}\vec{v}$ be the dyadic product of two spatial vectors, $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^3$, i.e. $A_{ij} = u_i v_j$.
   
   (a) Show that $\mathbf{A}$ is a second order tensor.
   
   (b) Show that $\text{det}(\mathbf{A}) = 0$.

3. Aris 2.44.2 (p.24)

4. Consider the vector operator $\nabla = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$.
   
   (a) Show that $\nabla \phi$ is a vector by the way it transforms under a coordinate change (where $\phi$ is a scalar function).
   
   (b) If $\mathbf{T}$ is a second order tensor, show that $\nabla \cdot \mathbf{T}$ is a vector.

5. Given a scalar function $\phi = \phi(x_1, x_2, x_3)$, do the quantities

   $$B_{ij} = \frac{\partial^2 \phi}{\partial x_i \partial x_j}$$

   form a second order tensor? Show why or why not.

6. Keener 4.1.5 (p.172)

7. Keener 4.1.6 (p.172)