1. Logan 1.2 #5 - transport-decay equation

2. Logan 1.7 #5 - Dirichlet problem

3. Logan 1.8 #4 - radial Laplace

4. Consider the PDE for $u(x, t)$:

$$u_{tt} + \alpha u_t = K u_{xx} \quad x \in \mathbb{R}$$

where $\alpha, K > 0$. Under certain “boundary” conditions at $\pm \infty$, there is an energy-like quantity which can be shown to be a non-increasing function of time.

   a) Find both the quantity and the required boundary conditions.

   b) Given that this looks like a wave equation, discuss the physical meaning of the result in (a).

5. Show that the PDE for $u(x, t)$:

$$-iu_t = u_{xx}$$

(where $i = \sqrt{-1}$) has travelling solutions of the form $u(x, t) = u(x - ct)$, with $c$ a constant.

6. Consider a conservation law equation for $u(x, t)$ in $\mathbb{R}^n$, with the flux function given as $\vec{f} = \nabla(\Delta u)$, and no sources or sinks. Derive a PDE for this case. What equation is obeyed by steady state solutions?