

Math 251H First Midterm Exam

75 minutes

February 20, 2009

**SAMPLE EXAM.**

1.(10 points) Find the general solution of the first order differential equation,

$$t^2y' - 2ty = 3.$$

2.(15 points) For the autonomous equation

$$y' = y(y - 2)(y + 4).$$

- a. (5 points) Find all equilibrium solutions.
- b. (5 points) Determine which of them are stable. Justify your answer.
- c. (5 points) Determine the behavior of solution  $y(t)$ , which satisfies the initial value  $y(1) = 1$ , when  $t \rightarrow +\infty$ .

**3.** (15 points) Consider the initial value problem:

$$6x^2 - 2xy + e^{x+y} + (e^{x+y} - x^2) \frac{dy}{dx} = 0, y(1) = -1.$$

- (a) (5 points) Verify that the equation is exact.  
(b) (10 points) Solve this initial value problem. You may leave your answer in implicit form.

4. (15 points) A tank initially contains 100 liters of pure water. A mixture containing a concentration of 5 grams/liter of salt enters the tank at a rate of 2 liters/min. and the well-stirred mixture leaves the tank at the same rate. Let  $Q(t)$  be the amount of salt in the tank. Formulate and state an initial value problem satisfied by  $Q$  modeling this process. Make sure you write down both an equation and an initial condition that  $Q(t)$  must satisfy.

5. (15 points) Find the solution of the following initial value problem using the method of undetermined coefficients:

$$y'' - 4y' + 3y = t^2 + te^{3t}, \quad y(0) = 1, \quad y'(0) = 0.$$

6. (10 points) Use the method of reduction of order to find a second solution of the differential equation:

$$t^2 y'' + 2ty' - 6y = 0, \quad t > 0,$$

knowing that  $y_1(t) = t^2$  is a solution. What is the **general** solution of the above equation?

7. (10 points) Find  $\alpha$  so that the solution to the initial value problem

$$y'' + 3y' - 4y = 0, y(0) = \alpha, y'(0) = 1,$$

converges to 0 as  $t \rightarrow +\infty$ .