MATH251H Practice Exam II

Spring 2008

120 minutes

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1. (15 points) Solve the following initial value problem,

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

2. (15 points) Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \mathbf{x}.$$

Classify the type and stability of the critical point at (0,0).

3. (20 points) Consider the nonlinear system,

$$\begin{cases} x' &= x - y \\ y' &= (x-1)(y-2). \end{cases}$$

a. Find all the critical points.

b. Linearize the system around each critical point, and classify its type and stability.

4. (20 points) Consider the following nonlinear system:

$$\begin{cases} x' &= -x^3 + xy^2 \\ y' &= -2x^2y - y^3 \end{cases}$$

Is the critical point at the origin stable?
(Hint: Consider the Liapunov function $V(x, y) = 2x^2 + y^2$.

5. (10 points) Solve the heat conduction equation,

$$\begin{cases} u_t &= 9u_{xx}, \quad 0 \le x \le 2, \\ u_x(0,t) &= 0 \\ u_x(2,t) &= 0 \\ u(x,0) &= \cos(\frac{3\pi}{2}x) - \cos(\pi x). \end{cases}$$

6. (20 points) Let

$$f(x) = \begin{cases} 2 - x & 0 \le x \le 2, \\ 2 + x & -2 \le x \le 0. \end{cases}$$

a. Compute the Fourier coefficients of $f(x)$ for $-2 \le x \le 2$.

b. Solve the initial-boundary value problem,

$$\begin{cases} u_{tt} &= 4u_{xx}, \quad 0 \le x \le 2, \\ u(0,t) &= 0 \\ u(2,t) &= 0 \\ u(x,0) &= f(x) \\ u_t(x,0) &= 0. \end{cases}$$

Express the solution as a Fourier series.