

**MATH 516: STOCHASTIC PROCESSES, Spring 2009****INSTRUCTOR:** Alexei Novikov, e-mail: [anovikov@math.psu.edu](mailto:anovikov@math.psu.edu)**TIME AND PLACE:** TTh 1:00-2:15pm 220 Willard**OFFICE HOURS:** T 2:30pm-3:30pm**webpage:** <http://www.math.psu.edu/~anovikov/math516/>**PREREQUISITES:** MATH416. **TEXT:** *notes* by L.C.Evans, and A.Dembo see webpage.**COURSE DESCRIPTION**

	<b>No. of lectures</b>
0. INTRODUCTION: Random walk & Finite state Markov chain (1 lecture) (E1)	1
1. CRASH COURSE IN PROBABILITY (5 lectures)	
1.1 Random variable, expectation, distribution function, independence (E2A-D)	1
1.2 Borel-Cantelli, Kolmogorov Inequality & Laws of Large Numbers (E2E,G)	1
1.3 Weak convergence of measures, Characteristic functions & CLT (E2F,G)	1
1.4 Conditional expectation (E2H, Dembo2)	1
1.5 Stochastic processes, Martingales (E2I)	1
2. BASIC STOCHASTIC PROCESSES (5 lectures)	
2.1 Filtrations, Stopping times. Poisson process (Dembo6.2)	1
2.2 Levy-Ciesielski construction of Brownian motion (E3B)	1
2.3 Finite state Markov and Markov jump processes (Dembo6.1, 6.3)	1
2.4 Finite-dimensional distributions. Kolmogorov consistency theorem.	1
2.5 Holder continuity of Brownian Motion. Markov Property (E3C,D)	1
3. STOCHASTIC INTEGRALS (4 Lectures)	
3.1 Quadratic variation (E4A)	1
3.2 Construction of the Ito integral (E4B)	1
3.3 Properties of the Ito integral (E4C)	1
3.4 Ito's formula (E4D,E)	1
4. STOCHASTIC DIFFERENTIAL EQUATIONS (3 Lectures)	
4.1 Definitions & examples. Linear SDEs (E5A,D)	1
4.2 Existence and uniqueness of solutions (E5B)	1
4.3 Properties of solutions.(E5C)	1
5. STOPPING TIMES (3 Lectures)	
5.1 Optional stopping theorem (E6A)	1
5.2 Doob's upcrossings lemma. Continuity of martingales.	1
5.3 Strong Markov property.	1
6. APPLICATIONS (9 Lectures)	
6.1 PDE's: Fokker-Plank, Backward Kolmogorov equations (E6B)	1
6.2 Optimal stopping (E6C)	1
6.3 Measures on $C([0, T])$ . Donsker's theorem Girsanov's transformation	1
6.4 Option pricing. (E6D)	1
6.5 Stochastic control of drift diffusion. Hamilton-Jacobi-Bellman equations	1
6.6 Ergodicity & mixing.	2
6.8 Jump-diffusion processes	1
6.9 Bayesian filtering	1
TOTAL PERIODS =	30

**HOMEWORKS:** Four homeworks due approximately every other Thursday, posted on the web. Specific dates are: Jan. 22, Feb. 5, Feb. 19, March 5**TAKE HOME MIDTERM** will be given March 26, due April 16.**COURSE GRADES:** Grades will be assigned on the basis of 400 points, distributed as follows:

Homeworks: 50 points each, Take home midterm: 200 points.

**ACADEMIC INTEGRITY** in *Policies and Rules, Student Guide to the University*, Policy 49-20.