1. Prove that in \( \mathbb{R}^n \) if \( \{x^k\}_{k=1}^\infty \) converges in topology induced by \( p \)-norm \( p \geq 1 \), then \( \{x^k\}_{k=1}^\infty \) converges in topology induced by (any other) \( q \)-norm \( q \geq 1 \). More formally, denote
\[
|x|_p = (|x_1|^p + |x_2|^p + \ldots + |x_n|^p)^{1/p}, \quad \text{if } 1 \leq p < \infty,
\]
\[
|x|_\infty = \max(|x_1|, |x_2|, \ldots, |x_n|).
\]
Suppose \( \{x^k\}_{k=1}^\infty \) \( x^k \in \mathbb{R}^n \), and there is \( x \in \mathbb{R}^n \) such that
\[
||x^k - x||_p \to 0, \quad \text{as } k \to \infty,
\]
for some \( p \geq 1 \). Prove that for any \( q \geq 1 \)
\[
||x^k - x||_q \to 0, \quad \text{as } k \to \infty,
\]
2. Construct an \( \epsilon \)-net for the set
\[
S = \{ x \in l_2, \sum_{n=1}^\infty n^2 x_n^2 \leq 1 \}.
\]
More specifically, explicitly show how for every \( \epsilon > 0 \) you can find \( N \) points \( x^1, x^2, \ldots, x^N \) so that for every \( y \in S \) we have that
\[
\min(||x^1 - y||_2, ||x^2 - y||_2, \ldots, ||x^N - y||_2) < \epsilon.
\]