Homework 7

1. Prove that in $\mathbb{R}^n$ an intersection of compact sets is a compact set.
2. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence in a compact set $K \subset \mathbb{R}^n$ that is not convergent. Show that there are two subsequences of this sequence that are convergent to different limit points of $K$.
3. Let $C$ be the Cantor set.
   a) Prove that $C$ has no interior points,
   b) prove that $C$ has no isolated points,
   c) compute the total length of the intervals removed from the unit interval in the construction of $C$ and compare it with the length of the interval,
   d) prove that $C$ is uncountable.
4. Describe all subsets of $\mathbb{R}^n$ that have no cluster (limit) points at all.
   Clarification: here you need to state the simplest (in your opinion) defining characteristic of these sets and show that if a set does not satisfy this property then it has a limit point.
   Reminder: A limit point of a set does not have to be in that set. For example the set $\{1/n\}$, $n \in \mathbb{N}$ has 0 as a limit point.