1. Suppose $a_n \geq 0$ for all $n \in \mathbb{N}$. Let

$$f_k = \{\text{number of } n \text{ such that } a_n > 1/k\}.$$ 

Prove that

$$\sum a_n < \infty \text{ if and only if } \sum f_n \frac{1}{n^2} < \infty.$$ 

2. Suppose

$$\sum a_n < \infty.$$ 

Let

$$S_N = \frac{1}{N} \sum_{n=1}^{N} na_n$$

Prove that

$$\lim_{N \to \infty} S_N = 0.$$ 

3. Let $f$ be a convex function on an open interval $I$. Prove that $f(x)$ is continuous on $I$.

Reminder $f(x)$ is convex on $I$ if for any $a, b \in I$ and any $t \in (0, 1)$ we have

$$f((1-t)a + tb) \leq (1-t)f(a) + tf(b).$$