1. Prove that if $X$ is a compact topological space, $Y$ is a metric space, then
\[ d(f, g) = \sup_{x \in X} \rho(f(x), g(x)) \]
is a metric on $C(X, Y)$.

2. **Strengthening of the Borel-Lebesgue thm** A collection of subsets $\mathcal{F}$ of a set $X$ is said to have a finite intersection property if for any finitely many $F_1 \in \mathcal{F}$, $F_2 \in \mathcal{F}$, $\ldots$, $F_n \in \mathcal{F}$ we have
\[ \bigcap_{i=1}^n F_i \neq \emptyset. \]
Prove that a topological space $X$ is compact if and only if for any collection of closed subsets $\mathcal{F} = F_\alpha | \alpha \in A$, that satisfies the finite intersection property we have
\[ \bigcap_{\alpha \in A} F_\alpha \neq \emptyset. \]
In other words compact sets are such that finite intersection property implies "infinite" intersection property. Note that here we do not assume that $X$ is a metric space.

3. Put a metric $\rho$ on all the words in a (English) dictionary by defining the distance between two (distinct) words to be $2^{-n}$ if the words agree for the first $n$ letters and are different at $(n + 1)$st letter. Here we agree that a space is distinct from a letter. For example, $\rho(\text{car}, \text{cart}) = 2^{-3}$ and $\rho(\text{car}, \text{call}) = 2^{-2}$.
(a) Verify that this is a metric,
(b) Suppose that words $w_1$, $w_2$ and $w_3$ are listed in alphabetical order. Show that
\[ \rho(w_1, w_2) \leq \rho(w_1, w_3). \]
(c) Suppose that words $w_1$, $w_2$ and $w_3$ are listed in alphabetical order. Find a formula for $\rho(w_1, w_3)$ in terms of $\rho(w_1, w_2)$ and $\rho(w_2, w_3)$.

4. Show that every compact metric space is separable, i.e. it has a countable dense subset.

5.(a) Prove that if $f$ is an bijective continuous function of a compact metric space $X$ onto a metric space $Y$, then $f^{-1}$ is continuous.
(b) Give an example when the statement is false if $X$ is not compact.

**Hint:** Map $[0, 1)$ onto a (unit) circle in $\mathbb{R}^2$. 

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