MATH 220 MIDTERM EXAMINATION March 4, 2003

Name______________________________ ID #_________________ Section #______

There are 20 multiple choice questions. Each problem is worth 5 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 12 PAGES (INCLUDING THIS ONE).

1. Consider

$$\begin{aligned} x_1 + 4x_2 + 3x_3 &= 1 \\ 2x_2 + x_3 &= 1 \\ x_2 + x_3 &= 1. \end{aligned}$$

The system of linear equations has

a) No solution.

b) Exactly one solution.

c) Infinitely many solutions.

d) Exactly two solutions.

2. Which of the following matrices is in **reduced** echelon form?

a) $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b) $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

c) $C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $D = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. Which of the following equations is linear in the variables x_1, x_2, and x_3?

a) $x_1^2 + x_2^2 + x_3^2 = 4$

b) $x_1x_2x_3 = 6$

c) $\sin(x_1)+\sin(x_2)+\sin(x_3) = 0$

d) $x_1 + (\ln\pi)x_2 + (\sqrt{\pi})x_3 = 2$

4. If $\begin{bmatrix} 1 & 1 & 2 \\ 2 & h & 3 \end{bmatrix}$ is the augmented matrix of a system of linear equations, then for what value of h is the system inconsistent?

a) $h = -1$

b) $h = 0$

c) $h = 1$

d) $h = 2$

5. If a given vector $\mathbf{b}$ is a linear combination of the columns of the matrix A=$[\mathbf{a}_1 \ \dots \ \mathbf{a}_n]$, then which of the following statements is **true**?

a) The equation $A\mathbf{x} = \mathbf{b}$ is consistent.

b) The equation $A\mathbf{x} = \mathbf{b}$ is inconsistent.

c) $\mathbf{b}$ coincides with one of the columns of A.

d) The set $\{\mathbf{a}_1, \dots, \mathbf{a}_n, \mathbf{b}\}$ is linearly independent.

6.

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 7 \\ 10 \\ 9 \end{bmatrix}$.

What is the geometric figure of Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

a) A point.

b) A line.

c) A plane.

d) All of $\mathbb{R}^3$.

7.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

The columns of which matrices span $\mathbb{R}^3$?

a) A and B.

b) A and C.

c) A only.

d) C only.

8.

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ 9 \\ 14 \end{bmatrix}$.

Which of the following statements is **true**?

a) $\mathbf{b}$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

b) $\mathbf{b}$ is not in Span$\{\mathbf{v}_1, \ \mathbf{v}_2\}$.

c) The set $\{\mathbf{v}_1, \mathbf{v}_2\}$ spans $\mathbb{R}^3$.

d) $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly dependent.

9. If $A = \begin{bmatrix} 1 & 0 & 3 & 0 & 3 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$ is the augmented matrix of a system of linear equations, then the solution set of the system in parametric vector form is

a) $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

b) $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$

c) $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

d) $\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

10. Which of the following statement is **true**?

a) A homogeneous system of linear equations is always consistent.

b) A homogeneous system of linear equations is always inconsistent.

c) A homogeneous system of linear equations always has nontrivial solutions.

d) If A is a 3×2 matrix, then A must have a column without a pivot position.

11.

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$ $\mathbf{v}_2 = \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$.

For what value(s) of h is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly **dependent**?

a) $h = 0$

b) All h

c) $h = -10$

d) $h = 8$

12. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation such that

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\5\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\6\end{bmatrix},$$

find $T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right)$.

a) $\begin{bmatrix}13\\7\end{bmatrix}$

b) $\begin{bmatrix}-7\\43\end{bmatrix}$

c) $\begin{bmatrix}-13\\-7\end{bmatrix}$

d) $\begin{bmatrix}10\\15\end{bmatrix}$

13. Which of the following formulae defines a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$?

a) $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_1\\2\end{bmatrix}$

b) $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}2x_1 + 5x_2\\x_1\end{bmatrix}$

c) $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}\sin(x_1)\\\cos(x_1)\end{bmatrix}$

d) $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}2 + x_1\\3 + x_1\end{bmatrix}$

14. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\-2\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}3\\1\end{bmatrix}.$$

Find $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

a) $\begin{bmatrix}-1\\5\end{bmatrix}$

b) $\begin{bmatrix}1\\5\end{bmatrix}$

c) $\begin{bmatrix}-1\\-5\end{bmatrix}$

d) $\begin{bmatrix}1\\-5\end{bmatrix}$

15. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation whose standard matrix is $A = \begin{bmatrix}1 & 0 & 1\\0 & 2 & 1\\1 & 0 & 1\end{bmatrix}$.

Which of the following statements is **true**?

a) T is both one-to-one and onto.

b) T is one-to-one, but not onto.

c) T is onto, but not one-to-one.

d) T is neither one-to-one nor onto.

18. If $A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, then

a) $(AB)^T = \begin{bmatrix} 6 & 7 \\ 9 & 8 \end{bmatrix}$

b) $(AB)^T = \begin{bmatrix} 6 & 9 \\ 7 & 8 \end{bmatrix}$

c) $(AB)^T = \begin{bmatrix} 7 & 6 \\ 8 & 9 \end{bmatrix}$

d) $(AB)^T = \begin{bmatrix} 9 & 8 \\ 6 & 7 \end{bmatrix}$

19. If A is a 10×7 matrix whose columns are *linearly independent*, then how many zero rows must the echelon form of A have?

a) 0

b) 3

c) 7

d) 10

20.

$$\text{Let } \mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ -1 \\ 2 \end{bmatrix}.$$

Which of the following vectors, when combinend with $\mathbf{u}$ and $\mathbf{v}$, makes a linearly independent set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

a) $\mathbf{w} = \begin{bmatrix} -2 \\ 3 \\ -1 \\ 1 \end{bmatrix}$

b) $\mathbf{w} = \begin{bmatrix} 11 \\ 1 \\ 3 \\ -1 \end{bmatrix}$

c) $\mathbf{w} = \begin{bmatrix} 0 \\ 7 \\ -1 \\ -1 \end{bmatrix}$

d) $\mathbf{w} = \begin{bmatrix} 9 \\ -3 \\ 3 \\ -6 \end{bmatrix}$