MATH 220

MIDTERM EXAMINATION

October 23, 2006

FORM A

NAME ______________________

STUDENT NUMBER ______________________

INSTRUCTOR ______________________

SECTION NUMBER ______________________

There are ??multiple choice questions in this examination. Each problem has four choices. Blacken only ONE oval for each problem. Each problem is worth 5 points.

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE ?? PROBLEMS ON ?? PAGES (INCLUDING THIS ONE).

1. Find all solutions of the following linear system:

$$\begin{array}{rcrcrcr} -x_1 & - & 2x_2 & + & 5x_3 & = & 1 \\ 2x_1 & + & 5x_2 & - & 8x_3 & = & -2 \\ x_1 & + & x_2 & - & 7x_3 & = & -1 \end{array}$$

a) $x_1 = -1, x_2 = 0, x_3 = 0$

b) $x_1 = 5x_3 - 1, x_2 = 2x_3$, and x_3 is free.

c) $x_1 = 9x_3 - 1, x_2 = -2x_3$, and x_3 is free.

d) There are no solutions.

2. Which of the following matrices is in reduced echelon form?

a) $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

3. If $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -7 & 0 \\ 0 & 2 & -6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, then describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has a solution.

a) $6b_1 + 2b_2 + b_3 = 0$

b) $6b_1 + 2b_2 + b_3 \neq 0$

c) $3b_1 + b_2 + b_3 = 0$

d) All values of b_1, b_2 and b_3

4. If $A = \begin{bmatrix} 1 & -2 & 4 & 3 \\ 1 & -1 & 7 & 8 \\ 3 & -5 & 15 & 14 \end{bmatrix}$, then which of the following best describes the geometric form of the set of all solutions of $A\mathbf{x} = \mathbf{0}$?

a) It is the zero vector.

b) A line.

c) A plane.

d) 3-dimensional space.

5. If $\begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & 7 & 10 & 6 \\ 1 & 5 & 8 & h \end{bmatrix}$ is the augmented matrix for a system of linear equations, then for which value(s) of h is the system consistent?

a) $h = 2$

b) $h = 6$

c) $h \neq 2$

d) $h \neq 6$

6. If $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$, then which of the following best describes the geometric form of the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

a) It is the zero vector.

b) A line.

c) A plane.

d) 3-dimensional space.

7. If A is an $m \times n$ matrix, and each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A, then which of the following statements is NOT always true?

a) For each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

b) The columns of A span $\mathbb{R}^m$.

c) A has a pivot position in every row.

d) The columns of A are linearly dependent.

8. Let $A = \begin{bmatrix} 1 & 3 & -7 \\ 0 & -1 & 4 \end{bmatrix}$, describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

a) $\mathbf{x} = x_3 \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$, with x_3 free.

b) $\mathbf{x} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

c) $\mathbf{x} = x_3 \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$, with x_3 free.

d) $\mathbf{x} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$

9. If A is an $m \times n$ matrix, with columns $\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_n$, and the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then which of the following statements is always true?

a) $\mathbf{b}$ is in $\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_n\}$.

b) The solution set is of the form $\text{span}\{\mathbf{v}_1, \ldots \mathbf{v}_p\}$.

c) A has a pivot position in every row.

d) The columns of A are linearly dependent.

10. Which of the following sets of vectors is linearly independent?

a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ 11 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

11. Which of the following formulae defines a linear transformation with domain $\mathbb{R}^2$ and codomain $\mathbb{R}^3$?

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 7x_2 \\ x_1 \\ -3x_1x_2 \end{bmatrix}$

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_1 \\ 3x_2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -9x_2 \\ x_1 - x_2 \\ -3x_1 \end{bmatrix}$

d) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 1 \\ x_2 + 7 \\ x_1 - x_2 \end{bmatrix}$

12. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(2\mathbf{e}_1) = 3\mathbf{e}_1 + 6\mathbf{e}_2$ and $T(-5\mathbf{e}_2) = 5\mathbf{e}_1 - 10\mathbf{e}_2$. What is the standard matrix for T? $\left(\text{Here } \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.\right)$

a) $\begin{bmatrix} 3 & 5 \\ 6 & 10 \end{bmatrix}$

b) $\begin{bmatrix} 3/2 & -1 \\ 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 6 \\ 5 & 10 \end{bmatrix}$

d) $\begin{bmatrix} 2/3 & -1 \\ 3 & 1/2 \end{bmatrix}$

13. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix $\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix}$. Which of the following vectors belongs to the range of T?

a) $\begin{bmatrix} 14 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ -7 \end{bmatrix}$

d) $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$

14. Let A be an invertible 3×3 matrix. Which of the following statements is FALSE?

a) The linear transformation with standard matrix A is one-to-one.

b) The solution set of the equation $A\mathbf{x} = \mathbf{0}$ consists of a single point.

c) For every nonzero vector $\mathbf{b}$ in $\mathbb{R}^3$, the solution set of the equation $A\mathbf{x} = \mathbf{b}$ spans $\mathbb{R}^3$.

d) The columns of A form a linearly independent set.

15. If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation whose standard matrix is $\begin{bmatrix} 1 & -4 \\ -2 & 9 \\ 2 & -8 \end{bmatrix}$, then which of the following statements is true?

a) T is one-to-one and onto.

b) T is one-to-one, but not onto.

c) T is neither one-to-one nor onto.

d) T is not one-to-one, but it is onto.

16. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$. What is A^2B?

a) $\begin{bmatrix} 6 & -3 \\ -10 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 8 & -4 \\ -16 & 8 \end{bmatrix}$

c) $\begin{bmatrix} -1 & -4 \\ 2 & 7 \end{bmatrix}$

d) $\begin{bmatrix} 6 & -3 \\ 14 & -7 \end{bmatrix}$

17. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$, then what is the first column of A^{-1}?

a) $\begin{bmatrix} -5 \\ 2 \\ -4 \end{bmatrix}$

b) $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} -5 \\ 3 \\ 1 \end{bmatrix}$

18. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}$, then what is $A^{-1} + B^{-1}$?

a) $\begin{bmatrix} -5 & 2 \\ -4 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 5/2 & -1 \\ 2 & -1 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix}$

d) $\begin{bmatrix} -2 & 2 \\ 0 & -1 \end{bmatrix}$