MATH 220 FINAL EXAMINATION December 13, 2001

Name_____ ID #_____ Section #_____

There are ??multiple choice questions. Each problem is worth 5 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE ?? PROBLEMS ON ?? PAGES (INCLUDING THIS ONE).

1. What is the solution of following system of equations

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ -2x_1 + x_2 - 2x_3 &= 3 \\ -x_1 + 3x_2 - x_3 &= 4? \end{aligned}$$

a) $x_1 = -1 - x_3, x_2 = 1, x_3$ is free.

b) $x_1 = 1 + x_2, x_2$ is free, $x_3 = -1$.

c) $x_1 = -x_2 + x_3$, and x_2, x_3 are free.

d) $x_1 = 2x_2 - x_3$, and x_2, x_3 are free.

2. Which of the following matrices is in echelon form (but not necessarily reduced echelon form)?

a) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 4 & 1 \\ 0 & 3 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \end{bmatrix}$

3. What geometric figure is formed from the span of the vectors $\begin{bmatrix}1\\2\\4\end{bmatrix}, \begin{bmatrix}5\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\11\\19\end{bmatrix}$?

a) a point.

b) a line.

c) a plane.

d) all of $\mathbb{R}^3$.

4. Which of the following sets of vectors span $\mathbb{R}^3$?

$$\text{A} = \left\{ \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}3\\1\\4\end{bmatrix} \right\} \qquad \text{B} = \left\{ \begin{bmatrix}2\\1\\3\end{bmatrix}, \begin{bmatrix}1\\3\\4\end{bmatrix} \right\} \qquad \text{C} = \left\{ \begin{bmatrix}2\\1\\0\end{bmatrix}, \begin{bmatrix}3\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix} \right\}$$

a) A only

b) B only

c) A and B

d) A and C

5. For what value(s) of h is the system with augmented matrix $\begin{bmatrix} 1 & 4 & -3 \\ 2 & h & -6 \end{bmatrix}$ consistent?

   a) It is consistent for all real h.

   b) It is consistent only if $h = 8$.

   c) It is consistent if $h \neq 8$.

   d) It is never consistent.

6. Suppose A is a matrix with three rows. Three of the following statements are equivalent. Which statement is not equivalent to the others?

   a) The columns of A span $\mathbb{R}^3$.

   b) A has exactly 3 pivots.

   c) A has a pivot in each column.

   d) $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b} \in \mathbb{R}^3$.

7. If the augmented matrix of a system of linear equations has the reduced echelon form shown below

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix},$$

then what is the set of solutions of the system?

a) $\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 3 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

b) $\mathbf{x} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

c) $\mathbf{x} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 3 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

d) $\mathbf{x} = \begin{bmatrix} 0 \\ -1 \\ 3 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}$

8. Find all values of h such that the set

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ h \\ 2 \end{bmatrix} \right\}$$

is linearly dependent.

a) 0

b) 1

c) 2

d) 3

9. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation which first reflects points in the x_1-axis and then rotates points in the counterclockwise direction through $\pi/2$ radians, then what is the standard matrix of T?

a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

10. If T is the linear transformation defined by the formula

$$T(x) = Ax, \qquad \text{where } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix},$$

then which of the following statements is true?

a) T is one–to–one and onto

b) T is one–to–one, but it is not onto

c) T is not one–to–one, but it is onto

d) T is neither one–to–one nor onto

11. If

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix},$$

then what is the second row of A^{-1}?

a) $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} -1 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$

12. Which of the following is a subspace of $\mathbb{R}^3$?

a) The null space of a 3×4 matrix.

b) The column space of a 4×3 matrix.

c) $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + x_2 \geq 0 \right\}$

d) $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : 4x_1 - x_2 = 0 \right\}$

13. If $A = \begin{bmatrix} 3 & -1 & 0 \\ 1 & 0 & 2 \\ 7 & -2 & 2 \end{bmatrix}$ then what is the dimension of the column space of A?

a) 0

b) 1

c) 2

d) 3

14. What is the determinant of the matrix $\begin{bmatrix} 1 & 5 & 2 & 9 \\ 0 & 0 & 8 & 29 \\ 0 & 0 & 0 & 5 \\ h & 3+5h & 5+2h & 11+9h \end{bmatrix}$?

a) 0

b) 60

c) 120

d) 180

15. If $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$, then what are the eigenvalues of A?

a) 2 and 3.

b) 3 and -7.

c) 5 and 4.

d) -2 and 3.

16. Suppose A and B are $n \times n$ matrices. Which of the following statements is **always** true?

a) If A and B are similar, then they have the same eigenvectors.

b) If A and B are similar, then $\det(A + B) = \det(A) + \det(B)$.

c) If A and B are similar, then they have the same eigenvalues.

d) If A and B are similar, then they are diagonalizable.

17. If λ is the eigenvalue of an $n \times n$ matrix A and $\mathbf{x}$ is the corresponding eigenvector, then which of the following statements is always true?

a) $\mathbf{x}$ is in the null space of $A - \lambda I$.

b) $\mathbf{x}$ is in the null space of A.

c) $\mathbf{x}$ is in the column space of $A - \lambda I$.

d) The column space of $A - \lambda I$ is all of $\mathbb{R}^n$.

18. If A, B and C are 3×3 matrices such that $\det(A) = 2, \det(B) = 3$ and $\det(C) = 4$, then what is $\det(2AB^TC^{-1})$?

a) 3

b) 6

c) 12

d) 3/2

19. If $A = \begin{bmatrix} -1 & 6 \\ -3 & 8 \end{bmatrix}$, then which of the following is a matrix P such that $P^{-1}AP$ is diagonal?

a) $P = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$,

b) $P = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$,

c) $P = \begin{bmatrix} -3 & 5 \\ 1 & 2 \end{bmatrix}$,

d) $P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

20. If $\mathbf{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then what is the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin?

a) $\sqrt{5}$

b) $2\sqrt{5}$

c) $3\sqrt{5}$

d) $4\sqrt{5}$

21. If W is the subspace of $\mathbb{R}^3$ spanned by the orthogonal vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and if $\mathbf{y} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, then what is the orthogonal projection of $\mathbf{y}$ onto W ?

a) $\mathbf{v}_1 - 2\mathbf{v}_2$

b) $2\mathbf{v}_1 - \mathbf{v}_2$

c) $2\mathbf{v}_1 + \frac{1}{2}\mathbf{v}_2$

d) $\mathbf{v}_1 + \frac{1}{2}\mathbf{v}_2$

22. if $\mathbf{x}_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$, and $\mathbf{x}_3 = \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$, then find an orthogonal set whose span is the same as the span of $\mathbf{x}_1, \mathbf{x}_2$ and $\mathbf{x}_3$.

a) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1/2\\0\\-1/2\\0\end{bmatrix}, \begin{bmatrix}1/4\\1/4\\1/4\\-3/4\end{bmatrix}$

b) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1/2\\-1/2\\1/2\\-1/2\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$

c) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}-3/4\\1/4\\1/4\\1/4\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$

d) $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1/2\\0\\-1/2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\-1\end{bmatrix}$

23. Find the least squares solution $\hat{\mathbf{x}}$ of $A\mathbf{x} = \mathbf{b}$ when $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

a) $\hat{\mathbf{x}} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

b) $\hat{\mathbf{x}} = \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$

c) $\hat{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

d) $\hat{\mathbf{x}} = \begin{bmatrix} 1/3 \\ -1/2 \end{bmatrix}$

24. What is an orthogonal matrix U that diagonalizes the symmetric matrix $\begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$?

a) $U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

b) $U = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -\sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$

c) $U = \frac{1}{2} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$

d) $U = \frac{1}{\sqrt{5}} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$

25. What is the matrix of the quadratic form $5x_1^2 - 8x_1x_2 - 7x_2^2$?

a) $\begin{bmatrix} 5 & -4 \\ -4 & -7 \end{bmatrix}$

b) $\begin{bmatrix} 5 & 4 \\ 4 & -7 \end{bmatrix}$

c) $\begin{bmatrix} 5 & -8 \\ -8 & -7 \end{bmatrix}$

d) $\begin{bmatrix} -5 & -8 \\ -8 & 7 \end{bmatrix}$