Math230 quiz No. 4 , Sep. 28, Name: , Student ID:

Announcement: Midterm I: Next Thursday, Oct.05, 2006, 6:30-7:45PM 102 Forum.

(1) (2 points) Find a vector equation and parametric equations for the line segment that joints P and Q:

$$P(-2,4,0), Q(6,-1,2)$$

Solution: $\vec{r}(t) = t\vec{r}_0 + (1-t)\vec{r}_1 \qquad 0 \le t \le 1$

$\vec{r}_0 = \vec{OP} = \langle -2, 4, 0 \rangle$

$\vec{r}_1 = \vec{OQ} = \langle 6, -1, 2 \rangle$

$\Rightarrow \vec{r}(t) = t\langle -2,4,0\rangle + (1-t)\langle 6,-1,2\rangle = \langle -2t, 4t, 0\rangle + \langle 6(1-t), -(1-t), 2(1-t)\rangle$

$= \langle 6-8t, 5t-1, 2-2t\rangle \quad 0\le t\le 1$ vector equation

parametric equations $\begin{cases} x = 6-8t, \\ y = 5t-1, \qquad 0 \le t \le 1 \\ z = 2-2t. \end{cases}$

(2) (2 points) Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point..

$$x = e^{-t}\cos t,\ y = e^{-t}\sin t,\ z = e^{-t};\ (1,0,1).$$

$\vec{r}(t) = \langle e^{-t}\cos t, e^{-t}\sin t, e^{-t}\rangle$

Solution: • tangent vector: $\vec{r}\,'(t) = \langle -e^{-t}\cos t - e^{-t}\sin t, -e^{-t}\sin t + e^{-t}\cos t, -e$

at the point $(1,0,1)$, corresponding parameter $t = 0$ (since $z = e^{-t} = 1$)

$\Rightarrow \vec{r}\,'(0) = \langle -1, 1, -1\rangle$: a direction vector of the tangent line at $(1,0$

$\Rightarrow$ tangent line at point $(1,0,1)$: $\langle x,y,z\rangle = \langle 1,0,1\rangle + t\langle -1,1,-1\rangle$

$= \langle 1-t, t, 1-t\rangle$

$\Rightarrow \begin{cases} x = 1-t \\ y = t \\ z = 1-t \end{cases}$

(3) (1 point) Find the curvature of the space curve

$$\vec{r}(t) = 3t\,\vec{i} + 4\sin t\,\vec{j} + 4\cos t\,\vec{k}$$

Solution: $\vec{r}\,'(t) = 3\vec{i} + 4\cos t\,\vec{j} - 4\sin t\,\vec{k}$

$|\vec{r}\,'(t)| = \sqrt{25} = 5$

$\vec{T}(t) = \frac{3}{5}\vec{i} + \frac{4}{5}\cos t\,\vec{j} - \frac{4}{5}\sin t\,\vec{k}, \quad \vec{T}\,'(t) = -\frac{4}{5}\sin t\,\vec{j} - \frac{4}{5}c$

$|\vec{T}\,'(t)| = \frac{4}{5}$

$\kappa = \dfrac{|T'(t)|}{|\vec{r}\,'(t)|} = \dfrac{\frac{4}{5}}{5} = \dfrac{4}{25}$