HW #5, Solution

Sec. 15.3.

16. $\frac{\partial z}{\partial x} = \frac{y}{x}$, $\frac{\partial z}{\partial y} = \ln x$

18. $f_x(x,y) = y x^{y-1}$, $f_y(x,y) = x^y \ln x$

20. $f_s(s,t) = \frac{2s^3 t}{(s^2+t^2)^2}$

37. $f_z(x,y,z) = -\frac{x}{(y+z)^2} \Rightarrow f_z(3,2,1) = -\frac{1}{3}$

41. $\frac{\partial z}{\partial x} = \frac{3yz-2x}{2z-3xy}$, $\frac{\partial z}{\partial y} = \frac{3xz-2y}{2z-3xy}$

50. $z_{xx} = 8y\sec^2(2x)\tan(2x)$

$z_{xy} = 2\sec^2(2x)$

$z_{yx} = 2\sec^2(2x)$

$z_{yy} = 0$

61. $\frac{\partial^3 u}{\partial r^2 \partial \theta} = \theta e^{r\theta}(2\sin\theta + \theta\cos\theta + r\theta\sin\theta)$

Section 15.4

4 $z = 4x - 4$

5. $z = y$

13. The partial derivatives are $f_x(x,y) = e^x(\cos xy - y\sin xy)$ and $f_y(x,y) = -xe^x \sin xy$

so $f_x(0,0) = 1$, $f_y(0,0) = 0$.

Both f_x and f_y are continuous, so f is differentiable at $(0,0)$

The linearization of f at $(0,0)$ is given by

$L(x,y) = x - 1$

14. $f_x(x,y) = \frac{1}{2}(x+e^{4y})^{-\frac{1}{2}}$ and $f_y(x,y) = 2e^{4y}(x+e^{4y})^{-\frac{1}{2}}$

So $f_x(3,0) = \frac{1}{4}$ and $f_y(3,0) = 1$

Both f_x and f_y are continuous functions near $(3,0)$, so f is differentiable at $(3,0)$

The linearization of f at $(3,0)$ is given by

$L(x,y) = \frac{1}{4}x + y + \frac{5}{4}$

17. The linear approximation of f at $(2,1)$ is given by

$f(x,y) \approx L(x,y) = -\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}$

$f(1.95, 1.08) \approx 2.846$

24. $dv = -y^2 \sin xy \, dx + (\cos xy - xy \sin xy) \, dy$

25. $du = e^t \sin\theta \, dt + e^t \cos\theta \, d\theta$

32. The surface area $S = 2(xy + yz + xz)$

$dS = 2(y+z)\,dx + 2(x+z)\,dy + 2(x+y)\,dz$

The maximum error occurs with $\Delta x = \Delta y = \Delta z = 0.2$

Using $dx = \Delta x$, $dy = \Delta y$, $dz = \Delta z$ we find the ~~max~~ maximum error in calculated surface area to be about $dS = 152 \text{ cm}^2$.