HW #4 Sol'ns

**14.4.** #13. $\vec{v}(t) = e^t \langle \cos t - \sin t,\ \sin t + \cos t,\ t+1 \rangle$

$\vec{a}(t) = e^t \langle -2\sin t,\ 2\cos t,\ t+2 \rangle$

$|\vec{v}(t)| = e^t \sqrt{t^2+2t+3}$

#16. $\vec{v}(t) = \vec{i} + \vec{j} - (10t+1)\vec{k}$

$\vec{r}(t) = (t+2)\vec{i} + (t+3)\vec{j} - (5t^2+t)\vec{k}$

#35. $a_T = e^t - e^{-t}$ $\left(= 2\sinh t\right)$

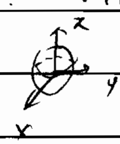
$a_N = \sqrt{2}$

**15.1.** #10 a) $g(2,-2,4) = 0$

b) $D = \{(x,y,z) \mid x^2+y^2+z^2 < 25\}$

c) range : $(-\infty, \ln 25)$

#19. $D = \{(x,y,z) \mid x^2+y^2+z^2 \le 1\}$. (The points inside or on the sphere of radius 1, center the origin)




#30 (a) VI, trace in $x=0$ is $z=|y|$, in $y=0$ is $z=|x|$

(b) V, trace in $x=0$ is $z=0$, in $y=0$ is $z=0$

(c) I " " " " $z=\frac{1}{1+y^2}$, " " " $z=\frac{1}{1+x^2}$

(d) IV " " " $z=y^4$, " " " $z=x^4$

(e) II " " " $z=y^2$, " " " $z=x^2$

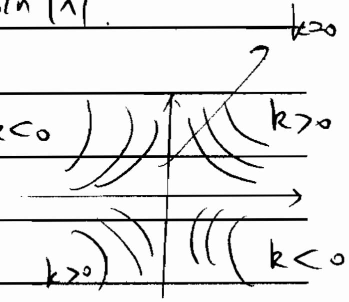
(f) III " " " $z=\sin|y|$, " " " $z=\sin|x|$.

#37. The level curves are $xy=k$,

$k=0$ : coordinate axis

$k>0$ : hyperbolas in the 1st and 3rd quardrants

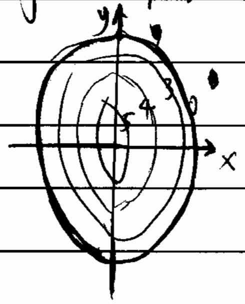
$k<0$ : " " " 2nd and 4th " "



#46. The contour map consists of the level curves $k = \sqrt{36-9x^2-4y^2}$

$\Rightarrow \; 9x^2+4y^2 = 36-k^2 \quad (k \geq 0)$

a family of ellipses with major axis the $y$-axis (or if $k=6$, the origin)




15.2. #6. $x-2y$ is a polynomial and therefore continuous

since $\cos t$ is a continuous function, the composition $\cos(x-2y)$ is also continuous

$xy$ is also a polynomial, and hence continuous, so the product $f(x,y) = xy\cos(x-2y)$

is a continuous function. Then $\lim\limits_{(x,y)\to(6,3)} f(x,y) = f(6,3) = 18$.

#9. On the $x$-axis: $f(x,0)=0$ for $x\neq 0$ so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along $x$-axis

$f(x,y) \xrightarrow[(x,y)\to(0,0)]{} \frac{1}{4}$ along the line $y=x$

Thus the limit doesn't exist.

#14. Use squeeze thm $\quad 0 \leq \dfrac{x^2\sin^2 y}{x^2+2y^2} \leq \sin^2 y$

$\downarrow$ as $(x,y)\to(0,0)$

$0$

So $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\sin^2 y}{x^2+2y^2} = 0$.

#20. $f(x,y,z) \xrightarrow[(x,y,z)\to(0,0,0)]{} 0$ along the $x$-axis

$f(x,y,z) \xrightarrow[(x,y,z)\to(0,0,0)]{} \frac{1}{2}$ along the line $y=x$, $z=0$.

Thus the limit doesn't exist.

#36. The first piece of $f$ is a rational function defined everywhere except at the origin, so $f$ is continuous on $\mathbb{R}^2$ except possibly at the origin. $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along the $x$-axis, $f(x,y)\to\frac{1}{3}$ as $(x,y)\to(0,0)$ along the line $y=x$

Thus $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ doesn't exist, so $f$ is not continuous at page $(0,0)$

and the largest set on which $f$ is continuous is $\{(x,y) \mid (x,y)\neq(0,0)\}$.