1 867 882

99a:99999 76-02 35Q30 35Q35 76B03 76D03 76D05

Majda, Andrew J. (1-NY-X; New York, NY);
Bertozzi, Andrea L. (1-DUKE; Durham, NC)

★**Vorticity and incompressible flow. (English summary)**

Cambridge Texts in Applied Mathematics, 27.
Cambridge University Press, *Cambridge*, 2002. xii+545 pp.
$110.00; $40.00 paperbound. ISBN 0-521-63057-6; 0-521-63948-4

The book is on incompressible flows that are inviscid or have high Reynolds numbers. The equations involved are the incompressible Euler equations for the inviscid cases (i.e., idealized materials) and the incompressible Navier-Stokes equations for viscous cases. These equations are physically important and mathematically challenging and interesting. They have stood the test of time and remain the most important in the field at present and for years to come. Physically their solutions are predictions of physical phenomena. Mathematically, their solutions are not yet proven to exist in the function class in which they are derived, notwithstanding numerous efforts by brilliant minds. One outstanding difficulty is associated with vorticity, i.e., the rotation (curl) of the velocity. Mathematically it seems that vorticity can become out of control everywhere, which corresponds to the physical phenomenon of turbulence. The control of vorticity lies at the heart of the regularity problem for these equations, and the regularity problem for the Navier-Stokes equations has recently been listed as one of the seven Millennium Prize Problems by the Clay Mathematics Institute (http://www.claymath.org/prize_problems/index.htm).

This long-awaited book is a "comprehensive introduction to the mathematical theory of vorticity and incompressible flow ranging from elementary introductory material to current research topics. Although the contents center on mathematical theory, many parts of the book showcase the interactions among rigorous mathematical theory, numerical, asymptotic, and qualitative simplified modeling, and physical phenomena" (from the preface).

The first half of the book can be used for an introductory graduate course on vorticity and incompressible flow. It contains many excellent exact and explicit solutions. It sets up the equations in various modern forms for further research. In particular, the approach used in Chapter 5 to search for singular solutions is novel. The work in Chapter 7 on interacting asymptotic filaments is also quite novel in many respects.

The second half can be used for a graduate course on the the-

ory of weak solutions for incompressible flow with an emphasis on modern applied mathematics. There are six chapters covering the following topics. The first is the elegant vortex patch theory in which Chemin's theorem (Theorems 8.7–8.8) is presented with a proof given by Bertozzi and Constantin. The next chapter covers the subtle theoretical and computational issues of vortex sheets, in particular Delort's theorem. The third chapter is on the concentration-cancellation phenomena in sequences of weak solutions. The fourth deals with the concept of measure-valued solutions. This chapter (Chapter 12) presents many open problems. The final chapter is on one-dimensional Vlasov-Poisson equations which serve as a simplified model for the two-dimensional Euler equations.

A key feature of the book is the natural and seamless incorporation of rigorous mathematical theory, numerical, asymptotic, and qualitative simplified modeling, and physical phenomena. This symbiotic interaction is apparent in 7 chapters out of 13.

This book is an outgrowth of several lecture courses by Majda, enriched by the authors' own research in the field and that of Majda's many students, collaborators and other researchers. Various students' lecture notes were in circulation and popular.

The chapters of the book are often supplemented with an appendix, as well as notes and references. The appendices are especially helpful for students. Interspersed reviews of topics such as singular integral operators are selected just right in presenting a complete and understandable exposition of the analysis. The book goes into deep mathematics without sacrificing much in the relative ease of understanding.

The book has a useful but short index. Maybe in the next edition the index can be expanded so that the terms Hausdorff, measure-valued solution, Kirchhoff, concentration-cancellation, etc., can be included, and indexed in each occurrence. There are inevitable typos, but this is a graduate level book and the reader should be able to discern the typos. The book has no exercises, but there are plenty of open problems. Topics such as incompressible limits, Hamiltonian/variational type structures, and boundary layers are not included.

There are about 11 books currently available on the market covering incompressible flows. Majda and Bertozzi's book is unique in covering both the classical and weak solutions for the incompressible and inviscid flows and it is excellently done.

Yu Xi Zheng (1-PAS; University Park, PA)