1. The stress tensor at a point has components given by

\[
(s_{ij}) = \begin{pmatrix}
2 & -1 & 2 \\
-1 & 3 & 0 \\
2 & 0 & -1
\end{pmatrix}.
\]

Find the stress vector \((p_n)\) across an area normal to the unit vector

\[n = (i_1 - i_2 + i_3)/\sqrt{3} \]

What is the normal stress across such an area (i.e., the projection \((p_n \cdot n)\) of the vector \(p_n\) onto \(n\))? 

2. Evaluate \( \oint_{|z-1|=1} \frac{1}{(z-1)^2} \, dz \). The direction of the contour is counterclockwise.

3. Find the adjoint operator \(T^*\) for the operator \(T\) defined on the Hilbert space \(L^2[a, b]\) by

\[Tu(x) = \int_{a}^{b} k(x)w(y)u(y)dy\]

where \(k(x) \in L^2([a, b])\) and \(w(y) \in C[a, b]\) are both given functions and \(u\) is an arbitrary member in \(L^2[a, b]\).

4. Show that the Fourier transform of \(f(x-c)\) is \(e^{i\mu c}\hat{f}(\mu)\).

5. Determine whether the zero solution to the system

\[
\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \epsilon \begin{pmatrix} y^2 \\ x^3 \end{pmatrix}
\]

where \(\epsilon\) is very small, is asymptotically stable or unstable. That is, determine whether all solutions with small data at \(t = 0\) go to zero or at least one solution with arbitrarily small data fails to go to zero as time goes to plus infinity. (The size of \(\epsilon\) depends on the size of the region of the initial data.)

6. Use the method of characteristics to derive a solution formula to

\[
\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} + cu = 0, \quad u(0, x) = g(x),
\]

where \(a\) and \(c\) are constants, \(t > 0\), and \(x \in \mathbb{R}^1\).
7. Find the fundamental solution $\phi$ for a dipole source:

$$\Delta \phi(x) = \nabla \delta(x) \cdot y \equiv \lim_{h \to 0} \frac{\delta(x + hy) - \delta(x)}{h},$$

where $y$ is a fixed unit vector in $\mathbb{R}^3$. (In electrostatics, a dipole source is like a battery with its two terminals extremely close together in physical space.)

8. Find a solution to

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad 0 < x < L, \ 0 < y < H$$

$u = 0$ on the boundary of the rectangle,

$u(0, x, y) = \sin \left( \frac{10\pi x}{L} \right) \sin \left( \frac{5\pi y}{H} \right) + \sin \left( \frac{20\pi x}{L} \right) \sin \left( \frac{4\pi y}{H} \right)$,

$\frac{\partial u}{\partial t}(0, x, y) = 0$. 

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