

M597K Table of Contents  
**Introduction to Applied Mathematics**

**Chapter I. Vectors and Tensors**

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- Appendix: Useful expressions

## **Chapter II. Complex Variables**

The three lectures cover the following sections of the text book by Keener.

- §6.1. Complex valued functions and branch cuts;
- §6.2.1. Differentiation and analytic functions, Cauchy-Riemann conditions;
- §6.2.2. Integration;
- §6.2.3. Cauchy integral formula;
- §6.2.4. Taylor series expansion.

## **Chapter III. Applied Functional Analysis**

- 3.1. Normed vector (linear) spaces of functions, Cauchy sequence, completeness, Banach and Hilbert spaces;
- 3.2. Bounded linear functional and operator; Riesz Representation theorem, adjoint operator,
- 3.3. Fredholm alternative theorem;
- 3.4. Spectral theory for compact operator.

## Chapter IV. Fourier and Laplace Transforms

- 4.1. Fourier integral transform, properties, examples;
- 4.2. Laplace transform, properties, examples.

## Chapter V. Ordinary Differential Equations

- 5.1 First-order linear scalar equation.
- 5.2 High-order linear scalar equation with constant coefficients.
- 5.3 First-order linear system with constant coefficients.
- 5.4 Stability of first-order linear system.
- 5.5 Hopf bifurcation.

## Chapter VI. Partial Differential Equations

### A. In infinite domains.

- 6.1. Transport equations, method of characteristics.
- 6.2. Wave equation in  $\mathbb{R}^1$ .
- 6.3. Wave equation in  $\mathbb{R}^3$ .
- 6.4. Wave equation in  $\mathbb{R}^2$ .
- 6.5. Heat equation in  $\mathbb{R}^n$  and  $\mathbb{R}_+^1$ .
- 6.6. Laplace and Poisson equations in  $\mathbb{R}^n$ .
- 6.7. Concept of fundamental solutions.

### B. On rectangular domains, separation of variables.

- 6.8. Laplace equation in a rectangle, Fourier series.
- 6.9. Poisson equation in a rectangle.
- 6.10. Heat equation in a rectangle.
- 6.11. Wave equation in a rectangle.
- 6.12. Eigenvalue problems, Sturm-Liouville operator.
- 6.13. Explicit eigenfunctions, orthogonal polynomials, special functions, Bessel's functions.

- 6.14. Vibrating circular membrane. Full Fourier series.

### C. General Bounded domains, Green's function.

- 6.15. Poisson equation in general bounded domains, Green's function.