### Introduction to Applied Mathematics

#### Chapter I. Vectors and Tensors

1.1. Vectors
   - 1.1.1. Projection of a vector
   - 1.1.2. Inner product
   - 1.1.3. Vector product
   - 1.1.4. Product of three vectors

1.2. Variable vectors
   - 1.2.1. Vector functions of a scalar argument
   - 1.2.2. The derivatives of a vector function
   - 1.2.3. The integral of a vector function

1.3. Vector fields
   - 1.3.1. Line integrals and circulation

1.4. Theorems of Gauss, Green, and Stokes
   - 1.4.1. Simply connected domains

1.5. Scalar fields
   - 1.5.1. Gradient
   - 1.5.2. Directional derivative

1.6. Vector fields
   - 1.6.1. Flux of a vector field
   - 1.6.2. The divergence of a vector field
   - 1.6.3. The curl of a vector field

1.7. Coordinate transformations

1.8. Zeroth-Order Tensors (Scalars)

1.9. First-Order (Cartesian) Tensors (Vectors)

1.10. Second-Order Tensors
   - 1.10.1. The Stress Tensor
   - 1.10.2. The moment of inertia tensor
   - 1.10.3. The Deformation Tensor
   - 1.10.4. The rate of Deformation Tensor
1.11. High-Order Tensors
1.12. Tensor Algebra
   1.12.1. Addition
   1.12.2. Multiplication
   1.12.3. Contraction of Tensors
1.13. Symmetry Properties of Tensors
1.14. Pseudotensors
1.15. Curvilinear Coordinate Systems
   1.15.1. Coordinate surfaces, coordinate curves, and local basis
   1.15.2. Arclength and orthogonal curvilinear coordinate systems
1.16. Grad, div, and curl in orthogonal curvilinear coordinate systems
   1.16.1. Gradient of a scalar field
   1.16.2. Divergence
   1.16.3. The curl

Appendix: Useful expressions

Chapter II. Complex Variables

The three lectures cover the following sections of the text book by Keener.
§6.1. Complex valued functions and branch cuts;
§6.2.1. Differentiation and analytic functions, Cauchy-Riemann conditions;
§6.2.2. Integration;
§6.2.3. Cauchy integral formula;
§6.2.4. Taylor series expansion.

Chapter III. Applied Functional Analysis

3.1. Normed vector (linear) spaces of functions, Cauchy sequence, completeness, Banach and Hilbert spaces;
3.2. Bounded linear functional and operator; Riesz Representation theorem, adjoint operator,
3.3. Fredholm alternative theorem;
3.4. Spectral theory for compact operator.
Chapter IV. Fourier and Laplace Transforms

4.1. Fourier integral transform, properties, examples;
4.2. Laplace transform, properties, examples.

Chapter V. Ordinary Differential Equations

5.1 First-order linear scalar equation.
5.2 High-order linear scalar equation with constant coefficients.
5.3 First-order linear system with constant coefficients.
5.4 Stability of first-order linear system.
5.5 Hopf bifurcation.

Chapter VI. Partial Differential Equations

A. In infinite domains.
   6.2. Wave equation in $\mathbb{R}^1$.
   6.3. Wave equation in $\mathbb{R}^3$.
   6.4. Wave equation in $\mathbb{R}^2$.
   6.5. Heat equation in $\mathbb{R}^n$ and $\mathbb{R}^1_+$.
   6.6. Laplace and Poisson equations in $\mathbb{R}^n$.
   6.7. Concept of fundamental solutions.

B. On rectangular domains, separation of variables.
   6.8. Laplace equation in a rectangle, Fourier series.
   6.9. Poisson equation in a rectangle.
   6.10. Heat equation in a rectangle.
   6.11. Wave equation in a rectangle.
   6.13. Explicit eigenfunctions, orthogonal polynomials, special functions, Bessel’s functions.

C. General Bounded domains, Green’s function.
   6.15. Poisson equation in general bounded domains, Green’s function.