M597K: Homework Assignment 9

Date: Wednesday Nov. 6, 2002; Due Wed. Nov. 13

1. Solve the initial value problem for a first-order linear homogeneous equation

$$\frac{dx}{dt} - (\sin t)\, x = 0, (t > 0); \quad x(0) = 1.$$

2. Solve the initial value problem for a first-order linear nonhomogeneous equation

$$\frac{dx}{dt} + (t+1)x = e^{-t^2}, (t > 0); \quad x(0) = 0.$$

3. Find the general solution formula for the second-order linear scalar equation with constant coefficients

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0.$$

4. Find the general solution formula for the first-order linear system of equations with constant coefficients

$$\begin{aligned} \tfrac{dx}{dt} &= 2x - y - z \\ \tfrac{dy}{dt} &= -x + 2y - z \\ \tfrac{dz}{dt} &= -x - y + 2z. \end{aligned}$$

5. Determine whether the zero solution to the system

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \epsilon \begin{pmatrix} y^2 \\ x^3 \end{pmatrix}$$

where ϵ is very small, is asymptotically stable or unstable. That is, determine whether all solutions with small data at $t = 0$ go to zero or at least one solution with arbitrarily small data fails to go to zero as time goes to plus infinity. (The size of ϵ depends on the size of the region of the initial data.)

You do not need to turn in the following problems which are more advanced and optional.

Optional 1. Find the general solution formula to the equation

$$\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 0.$$

Is the zero solution asymptotically stable?

Optional 2. Find the general solution formula to the system of equations

$$\frac{dx_1}{dt} = x_2 - x_3, \quad \frac{dx_2}{dt} = x_1 + x_2, \quad \frac{dx_3}{dt} = x_1 + x_3.$$