

# M597K: Homework Assignment 8

Date: Oct. 30, Wed.; Due Wed. Nov. 6.

1. Follow the proof of Property b to prove

$$(f * g)^\vee(\mu) = \sqrt{2\pi} f^\vee(\mu) \cdot g^\vee(\mu).$$

2. Find the Fourier transform of

$$f(x) = \begin{cases} 0, & x < 0, \text{ or } x > a, \\ 1, & 0 \leq x \leq a. \end{cases}$$

3. Show that the Fourier transform of  $f(x)e^{icx}$  is  $\hat{f}(\mu + c)$ . Here  $c$  is a real number.  
4. Use Fourier transform to solve the heat (diffusion) equation with reaction:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + cu, \quad (x \in \mathbb{R}^1)$$

where  $k > 0$  and  $c$  is a real number, with initial condition

$$u(x, 0) = f(x).$$

5. Find the Laplace transform of the function  $f(t) = t^2 + e^t, t \geq 0$ .  
6. Use Laplace transform to solve

$$\begin{aligned} u'' + 2u' + u &= e^{-t} \quad \text{for } t > 0, \\ u(0) &= 0, \\ u'(0) &= 0. \end{aligned}$$

7. Use Laplace transform to solve

$$\begin{aligned} u'' + 2u' - 3u &= 0 \quad \text{for } t > 0, \\ u(0) &= 1, \\ u'(0) &= -2. \end{aligned}$$

(Seven problems in all, about 14.4 points each.)