M597K: Homework Assignment 7

Date: Oct. 16, Wed., 2002. Due Friday Oct. 25.

The following problems are on the specified pages of the text book by Keener (2nd Edition, i.e., revised and updated version)

Problems 3 and 4 of Section 2.1 on p.94;

Problem 1 of Section 3.1 on p.128;

Problem 1 of Section 3.2 on p.128.

5. Let ℓ^2 denote all the sequences $(x_1, x_2, x_3, \cdots, x_n, \cdots)$ of real numbers. Let x denote such a sequence. Use vector addition and scalar multiplication. Then it is a vector space (no proof needed). Use the inner product

$$< x, y > = \sum_{i=1}^{\infty} x_i y_i$$

where $y = (y_1, y_2, \cdots, y_n, \cdots)$. Show that this inner product is well defined in ℓ^2 and it satisfies the four properties of the definition of inner product. (It is called "the little ℓ two space". (Reference: p. 59 of text book)

6. Let $f(t)$ be equal to 100 for t between 19 and 20, and equal to zero for all other times t. This may represent a Miss Universe's status score history of her lifetime. Let $g(t)$ be equal to 85 for t between 35 and 116, and equal to zero for all other times t. This might represent another person's social status score history, who at age 35 invented a perpetual machine and enjoyed the fame he got throughout his lifetime. (Unfortunately he lived only to 116. Thus his perpetual machine was just somewhat perpetual, and that explains the score 85 instead of a higher number.) Now a panel want to select one from the two to put into a Hall of Fame and want you to give the panel a single measurement number from each of the two so that the panel can decide. Do the maximum norm and the L^2 norm calculation (serious part) and decide which norm you want to use to give to the panel (a decision that is purely up to you).

7. Solve Problem 5, p. 94, of the text book, where L^2 is replaced by L^1.

8. Show that the functional T on the Banach space $C[0, 1]$ defined by

$$Tf = \int_0^1 \frac{f(x)}{\sqrt{x}} dx, \quad f(x) \in C[0, 1]$$

is linear and bounded.

9. Let a and b be two points in the interval $[0,1]$. Show that the functional T on the Banach space $C[0,1]$ defined by

$$Tf = f(a) - f(b), \quad f(x) \in C[0,1]$$

is linear and bounded. (This functional is generally written as $\delta(x-a) - \delta(x-b)$.) (Hint: Lecture notes might help.)

10. Find the adjoint operator T^* for the operator T defined on the Hilbert space $L^2[a,b]$ by

$$Tu(x) = w(x) \int_a^b k(x,y)u(y)dy$$

where $k(x,y) \in L^2([a,b] \times [a,b])$ and $w(x) \in C[a,b]$ are both given functions and u is an arbitrary member in $L^2[a,b]$. (Hint: Read text book p.107.)

11. Verify that $\lambda = (n\pi)^2$ and $u = \cos(n\pi x)$ are eigenvalues and corresponding eigenfunctions for the Sturm-Liouville eigenvalue problem:

$$\frac{d^2u}{dx^2} + \lambda u = 0, (0 < x < 1); \quad u'(0) = u'(1) = 0$$

for all positive integer n.