# M597K: Homework Assignment 6

Date: October 2, Wednesday, 2002. Due Friday. Oct. 11.

**1.** Find the numerical value of each of the following and express it in the form $a+bi$ where $a$ and $b$ are real numbers.

(a) $i(1+i)(2+i)$,

(b) $(1-i)^2+(3-2i)^2$,

(c) $i^{2002}$.

**2.** Find the polar representation ($re^{i\theta}$) for each of the following

(a) $1-i$,

(b) $(1+i)/(1+\sqrt{3}i)$.

**3a.** From $e^{i\theta}=\cos\theta+i\sin\theta$ and $e^{-i\theta}=\cos\theta-i\sin\theta$, one can derive

$$\cos\theta=\frac{1}{2}(e^{i\theta}+e^{-i\theta}),\quad \sin\theta=\frac{1}{2i}(e^{i\theta}-e^{-i\theta}).$$

Then define

$$\cos z=\frac{1}{2}(e^{iz}+e^{-iz}),\quad \sin z=\frac{1}{2i}(e^{iz}-e^{-iz})$$

for all complex $z$. Find the derivatives $\cos' z$ and $\sin' z$.

**3b.** From $\sinh\theta=\frac{1}{2}(e^{\theta}-e^{-\theta})$ and $\cosh\theta=\frac{1}{2}(e^{\theta}+e^{-\theta})$, we define $\sinh z$ and $\cosh z$ for all complex $z$ by replacing the real $\theta$ by $z$. Find the derivatives $\sinh'' z$ and $\cosh'' z$.

**4.** Describe the range of the function

$$w=\ln z=\ln r+i\theta \qquad \text{for } z=re^{i\theta},\quad \theta\in(-\pi,\pi)$$

with the domain $\Omega=\{z=re^{i\theta}\mid r>0,-\pi<\theta<\pi\}$. What is the image of a circle $|z|=c>0$? What is the image of a ray $\theta=\alpha$?

**5.** Use the Cauchy-Riemann conditions to verify that the function

$$f(z)=\frac{1}{2}\ln(x^2+y^2)+i\arctan\frac{y}{x},$$

where $z=x+iy$, $-\pi<\arctan\frac{y}{x}\le\pi$ is analytic in the right half plane $x>0$.

**6.** Is $f(z)=\bar{z}z$ (where bar denotes complex conjugate) analytic in the entire plane?

**7.** Find the Taylor series for $f(z)=\frac{1+z}{z^2-1}$ at $z_0=0$.

**8.** Find the complex integral $\int_C \bar{z}\, dz$ where $C$ is the arc $y = x^3$ for $0 \leq x \leq 1$. The direction of integration is from $x = 0$ to $x = 1$.

**9.** Find $\int_C (z^n + z + 1)\, dz$ where $n \geq 1$ is an integer and $C$ is a closed contour.

**10.** Evaluate $\oint_{|z-1|=1} \frac{z+1}{(z-1)^2}\, dz$. The direction of the contour is counterclockwise. Note that the symbol $|z-1| = 1$ denotes the circle

$$(x-1)^2 + y^2 = 1.$$