M597K: Homework Assignment 2

Date: Wed., Sept. 4, 2002. Due Friday September 13.

1. Find the derivative of the vector $\mathbf{A}(t) = (\cos t, \sin t, 2t)$. Draw the graph of $\mathbf{A}(t)$ with $\mathbf{A}'(t)$. (Hand drawing is ok.)

2. Integrate the vector $\mathbf{B}(t) = (e^t, \sin t, 2t)$ to find $\int_0^1 \mathbf{B}(t)\, dt$. (All numbers are in radian, not degree.)

3. Evaluate the line integral

$$\int_L \mathbf{C} \cdot d\mathbf{r}$$

where $\mathbf{C} = (x_2, -x_1, -1)$ and L is a directed curve given by the graph of the vector $\mathbf{A}(t)$ in Exercise 1 from $t = 0$ to $t = 2\pi$.

4. Find the total circulation

$$\oint_C (x_1 + x_2)dx_1 + (x_1 - x_2)dx_2$$

where C is the ellipse $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$ going counter-clockwise.

5. Find the gradient of $f = x_1^2 + x_2^2 + x_3^2$.

6. Find a unit normal vector to the surface

$$x_3 = 2 - x_1 - x_2^2.$$

7. Find the divergence and curl of the vector field

$$\mathbf{A} = (x_2 x_3, x_1 x_3, x_1 x_2).$$

8. Find the total flux of the vector field $\mathbf{A} = (x_1, x_2, x_3)$ out of the unit sphere: $x_1^2 + x_2^2 + x_3^2 = 1$.

9. Evaluate the line integral by using Green's theorem:

$$\oint_C (x^2 + y^2)dx + 2xydy,$$

where C is the square bounded by the lines $x = 0, x = 2, y = 0, y = 2$.

10. Let

$$\mathbf{F}(x_1, x_2, x_3) = (x_1 \mathbf{i}_1 + x_2 \mathbf{i}_2 + x_3 \mathbf{i}_3)/r^3$$

where $r^2 = x_1^2 + x_2^2 + x_3^2$. Show that the flux of this vector through any closed surface S is 0 if the origin is not enclosed by S.