

M597K: Homework Assignment 14 (Last One)

Date: Dec 9, Monday; Due by Monday Dec. 16

1. Use Rayleigh quotient (Section 6.12, Conclusion No. 6) to show that any eigenvalue must be positive ($\lambda > 0$) for Bessel's equation

$$\begin{cases} (ru')' - \frac{m^2}{r}u + \lambda ru = 0, & 0 < r < a, \\ u(a) = 0, \\ |u(0)| < \infty, |u'(0)| < \infty. \end{cases} \quad (1)$$

(Hint: Once you use the Rayleigh quotient and the boundary conditions, the positivity of λ_n is obvious.)

2. Use separation of variables to solve the two-dimensional eigenvalue problem for the Laplacian in a rectangle

$$\begin{cases} u_{xx} + u_{yy} + \lambda u = 0, & \text{in } 0 < x < L, 0 < y < H, \\ u = 0 \text{ on the boundary.} \end{cases} \quad (2)$$

(The eigenfunctions should be the eigenfunctions of Section 6.9.)

END