M597K: Homework Assignment 13

Date: Dec. 2, Monday; Due Wed. Dec. 11.

1. Use change of variables to reduce

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2} + Q(t,x), \qquad 0 < x < L, \\ u(0,x) &= g(x), \\ \frac{\partial u}{\partial x}(t,0) &= A(t), \\ \frac{\partial u}{\partial x}(t,L) &= B(t) \end{aligned} \tag{1}$$

to a problem with homogeneous boundary condition.

2. Find a solution to

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \qquad 0 < x < L,\ 0 < y < H \\ u &= 0 \quad \text{on the boundary of the rectangle}, \\ u(0,x,y) &= 5\sin\left(\frac{7\pi x}{L}\right)\sin\left(\frac{11\pi y}{H}\right) + 3\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{\pi y}{H}\right), \\ \frac{\partial u}{\partial t}(0,x,y) &= 0. \end{aligned}$$

3. Use eigenfunction expansion to derive a solution to

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + Q(t,x,y), \qquad 0 < x < L,\ 0 < y < H \\ u &= 0 \quad \text{on the boundary of the rectangle}, \\ u(0,x,y) &= 0, \\ \frac{\partial u}{\partial t}(0,x,y) &= 0. \end{aligned}$$