

M597K: Homework Assignment 10

Date: Nov. 11, Monday; Due Wed. Nov. 20.

1. Use the method of characteristics to derive a solution formula to

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} + cu = 0, \quad u(0, x) = g(x),$$

where a and c are constants, $t > 0$, and $x \in \mathbb{R}^1$. (Hint: Derive an ordinary differential equation for u along a characteristic curve $x = x_0 + at$.)

2. Use the appropriate formula to find the solution $u(t, x)$ to

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - 4 \frac{\partial^2 u}{\partial x^2} = 0, \\ u(0, x) = e^{-x^2}, \\ \frac{\partial u}{\partial t}(0, x) = 0. \end{cases}$$

3. Find the value at $x = 0, t = 2$ of the solution of the initial value problem

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = x, \\ u(0, x) = 0, \\ \frac{\partial u}{\partial t}(0, x) = 0. \end{cases}$$

4. In the three dimensional homogeneous wave equation ($f = 0$) with speed $c = 1$, let $u(0, x_1, x_2, x_3) = 0, \frac{\partial u}{\partial t}(0, x_1, x_2, x_3) = h(x_1, x_2, x_3)$. Suppose that $h(x_1, x_2, x_3) = 0$ for $x_1^2 + x_2^2 + x_3^2 \geq 1$. Show or explain that $u(t, 0, 0, 0) = 0$ for all $t > 1$.

5. In the two dimensional homogeneous wave equation ($f = 0$) with speed $c = 1$, let $u(0, x_1, x_2) = 0, \frac{\partial u}{\partial t}(0, x_1, x_2) = h(x_1, x_2)$. Suppose that $h(x_1, x_2) = 0$ for $x_1^2 + x_2^2 \geq 1$ and $h(x_1, x_2) > 0$ for $x_1^2 + x_2^2 < 1$. Show or explain that $u(t, 0, 0) > 0$ for all $t > 0$.

- Optional 6.** Use the method of characteristics to solve the problem

$$\begin{cases} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0, & t > 0, x \in \mathbb{R}, \\ u(0, x) = e^x. \end{cases}$$

====End====