M597K: Homework Assignment 1

Date: August 28, 2002. Due Friday Sept 6.

1. Let $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ be a basis (may or may not be orthonormalized). Let $\mathbf{A}$ be a vector. Is it always true that

$$\mathrm{Proj}_{\mathbf{e}_1}\mathbf{A} + \mathrm{Proj}_{\mathbf{e}_2}\mathbf{A} + \mathrm{Proj}_{\mathbf{e}_3}\mathbf{A} = \mathbf{A}?$$

2. A parallelogram has acute angle $\pi/4$ and side lengths $a = 2, b = 5$. Thinking of the corresponding sides as vectors $\mathbf{a}$ and $\mathbf{b}$, find

(a) The vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ (what is their geometric meaning?);

(b) The area of the parallelogram.

3. Given the vectors

$$\mathbf{A} = \mathbf{i}_1 + 2\mathbf{i}_2 + 3\mathbf{i}_3, \qquad \mathbf{B} = 4\mathbf{i}_1 - 5\mathbf{i}_2 - 2\mathbf{i}_3,$$
$$\mathbf{C} = 3\mathbf{i}_1 + 2\mathbf{i}_2 + \mathbf{i}_3, \qquad \mathbf{D} = \mathbf{i}_1 + 3\mathbf{i}_2 + 4\mathbf{i}_3;$$

where $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ are an orthonormal basis. Find

(a) $\mathbf{A} - \mathbf{B} + 2\mathbf{C}$;

(b) $\mathbf{A} \cdot \mathbf{B}$;

(c) The angle made by $\mathbf{C}$ and $\mathbf{D}$;

(d) The projection of $\mathbf{A}$ onto the direction of $\mathbf{B}$;

(e) The vector product $\mathbf{C} \times \mathbf{D}$.

4. Show that the four vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ given in the previous problem are linearly dependent.

5. Use the fomulas learned in class (lecture notes) to verify the following identities:

$$\begin{aligned}(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) &= \mathbf{b}[\mathbf{a} \cdot (\mathbf{c} \times \mathbf{d})] - \mathbf{a}[\mathbf{b} \cdot (\mathbf{c} \times \mathbf{d})] \\ &= \mathbf{c}[\mathbf{a} \cdot (\mathbf{b} \times \mathbf{d})] - \mathbf{d}[\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})].\end{aligned}$$